(C) SIMPLE APPLICATIONS OF REFRACTION OF LIGHT

4.10 REAL AND APPARENT DEPTH

An object placed in a denser medium when viewed from a rarer medium, appears to be at a depth lesser than its real depth. This is because of refraction of light.

In Fig. 4.36, consider a point object O kept at the bottom of a transparent medium (such as water or glass) separated from air by the surface PQ. A ray of light OA, starting from the object O, is incident on the surface PQ normally, so it passes undeviated along the path AA′. Another ray OB, starting from the object O, strikes the boundary surface PQ at B and suffers refraction. Since the ray travels from a denser medium (water or glass) to a rarer medium (air), so it bends away from the normal N′BN drawn at the point of incidence B on the surface PQ and travels along BC in air.

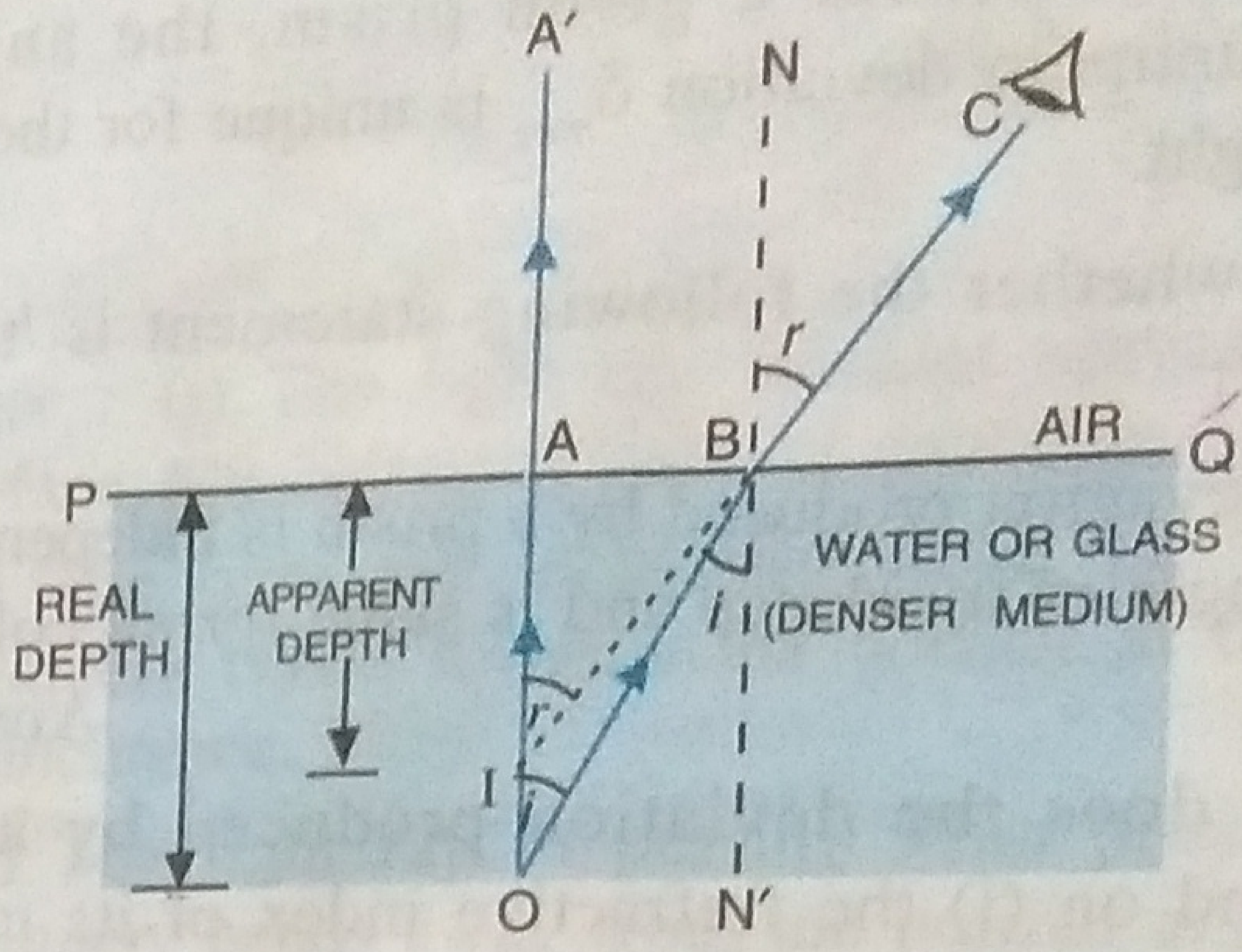

Fig. 4.36 Real and apparent depth

> *Note :* The point B is very close to the point A, and both the rays OA′ and BC enter the eye simultaneously. In Fig. 4.36, they have been shown separately for the sake of clarity of the ray diagram.

When viewed by the eye, the ray BC appears to be coming from a point I which is the *virtual image* of O, obtained on producing A′A and CB backwards. Thus any object (*e.g.* a coin) placed at O, when seen from above (air), will appear to be at I which is at a lesser depth (= AI) than its actual depth (= AO).

In Fig. 4.36, for the incident ray OB, angle of incidence $i = \angle OBN'$ and angle of refraction $r = \angle CBN$. Since AO and BN′ are parallel and OB is a transversal line, so

$$\angle AOB = \angle OBN' = i$$

Similarly, IA′ and BN are parallel and IC is the transversal line, so

$$\angle BIA' = \angle CBN = r$$

Now in right-angled triangle BAO,

$$\sin i = \frac{BA}{OB}$$

and in right-angled triangle IAB,

$$\sin r = \frac{BA}{IB}$$

For refraction from medium (water or glass) to air, by Snell's law

$$\frac{\sin i}{\sin r} = {}_m\mu_a$$

or $${}_m\mu_a = \frac{\sin i}{\sin r} = \frac{BA/OB}{BA/IB} = \frac{IB}{OB}$$

∴ Refractive index of medium with respect to air

$${}_a\mu_m = \frac{1}{{}_m\mu_a} = \frac{OB}{IB}$$

Since the point B is very close to the point A, *i.e.*, the object is viewed from a point vertically above the object O, ∴ IB = IA and OB = OA.

Hence $${}_a\mu_m = \frac{OA}{IA} = \frac{\text{real depth}}{\text{apparent depth}} \quad ...(4.13)$$

or $$\text{Apparent depth} = \frac{\text{real depth}}{{}_a\mu_m} \quad ....(4.14)$$

Thus,

Shift OI = real depth – apparent depth

or $$\text{Shift} = \text{real depth} \times \left(1 - \frac{1}{{}_a\mu_m}\right) \quad ..(4.15)$$

The shift by which the object appears to be raised, depends on :

(1) *the refractive index of the medium,*

(2) *the thickness of the denser medium, and*

(3) *the colour* (or wavelength) *of incident light*.*

(1) Dependence of shift on the refractive index : Higher the refraction index of the medium, more is the shift.

(2) Dependence of shift on the thickness of medium : Thicker the medium, more is the shift.

(3) Dependence on wavelength (or colour) of light : The shift decreases with the increase in the wavelength of light used. Since $\mu_V > \mu_R$, therefore the shift is more for the violet light than for the red light in a given medium.

Examples

(i) For glass, ${}_a\mu_g = \frac{3}{2}$, therefore the thickness of glass slab appears only two-third of its real thickness when it is viewed from air by keeping the eye vertical above the slab.

(ii) For water, ${}_a\mu_w = \frac{4}{3}$, therefore the depth of a water pond appears three-fourth of its real depth on seeing it from air in a nearly vertical direction (*i.e.*, it appears shallow). This is why a fish when seen from air appears to be nearer the surface of water than at its actual depth.

> *Note :* The apparent depth of an object lying in a denser medium is always less than its real depth when viewed from any direction in the rarer medium. But the above eqns. (4.13), (4.14) and (4.15) are valid only when the object is seen from vertically above.

* *The refractive index of a medium increases with the decrease in wavelength of incident light* ($\mu_V > \mu_R$).

4.11 APPARENT BENDING OF A STICK UNDER WATER

Fig. 4.37 shows a straight stick (or pencil) XOP placed obliquely in water. The portion OP of the stick (or pencil) under water when seen from air appears to be shortened and raised up as OP′. This is due to refraction of light from water to air. The rays of light coming from tip P of the stick (or pencil) when pass from water to air, bend away from the normal and appear to be coming from a point P′ which is the *virtual image* of the point P. The same is true for every point of the stick (or pencil) inside water from P to O.

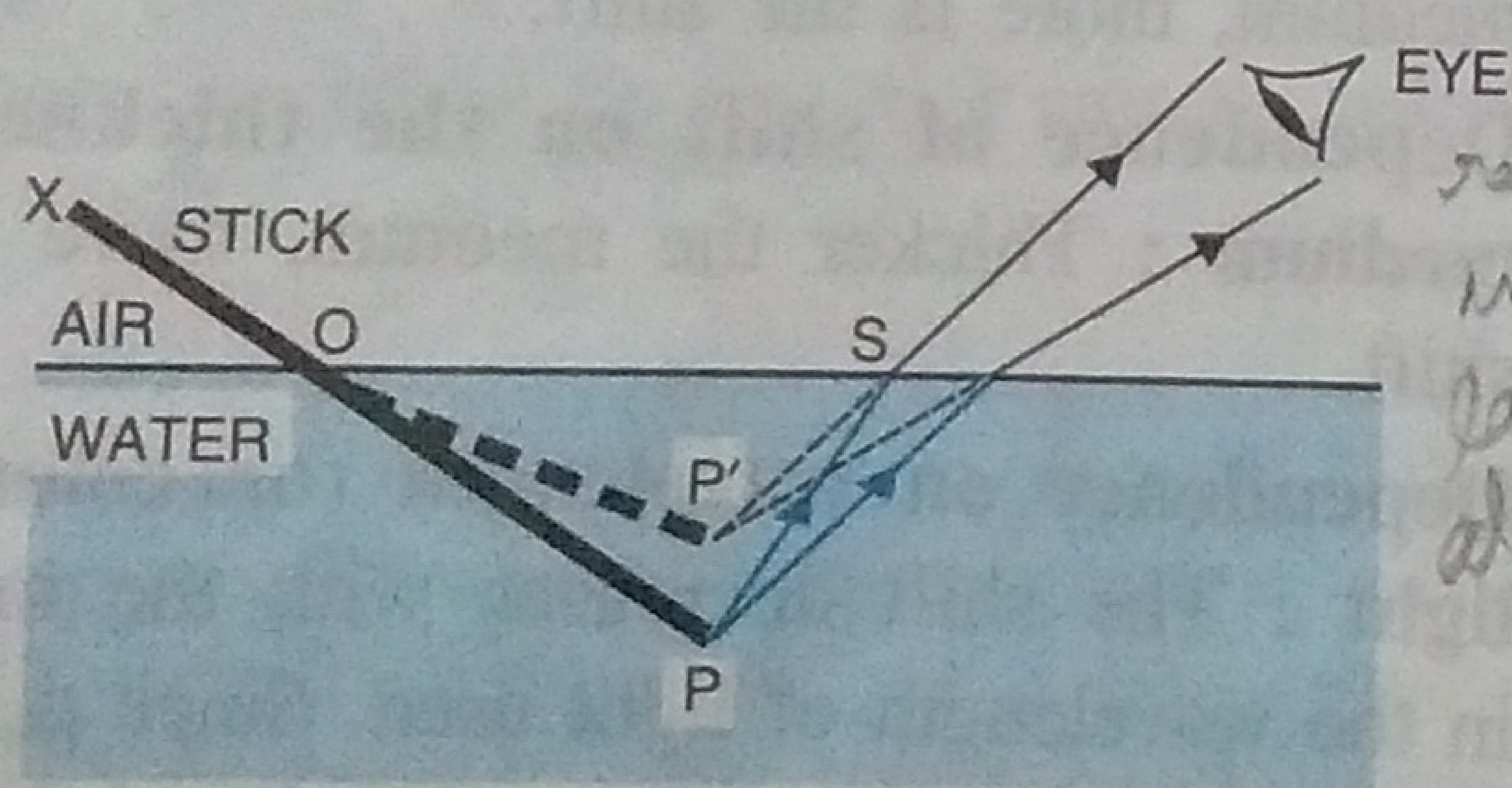

Fig. 4.37 Bending of stick due to refraction

Thus, the part PO of the stick (or pencil) appears to be P′O, *i.e., the immersed part of the stick appears to be raised and therefore bent at the point O on the surface of water and the stick (or pencil) XOP appears as XOP′.*

> ***Note :** An object placed in a rarer medium when viewed from a denser medium appears to be at a greater distance than its real distance. In Fig. 4.38, an object O placed in air when viewed from water appears to be at I which is higher than the object O.*

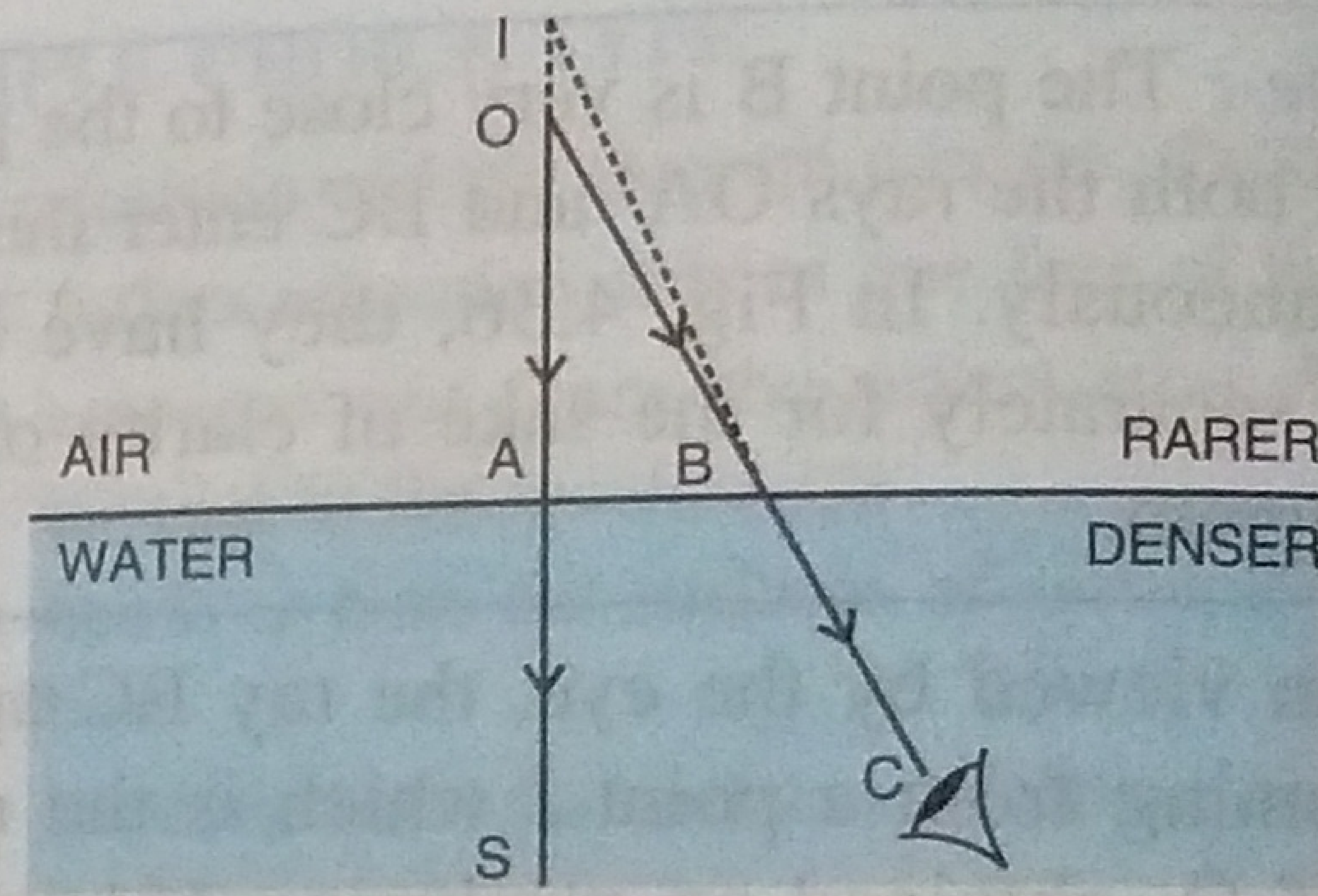

Fig. 4.38 An object in rarer medium viewed from a denser medium

4.12 SOME CONSEQUENCES OF THE REFRACTION OF LIGHT

In our daily life we come across many phenomena which are caused by the refraction of light. Some of these are given below :

(i) *A star appears twinkling in the sky due to change in refractive index of air with temperature.*

(ii) *The sun is seen a few minutes before it rises above the horizon in the morning while in the evening few minutes longer after it sets.*

(iii) *A coin kept in a vessel and not visible when seen from just below the edge of the vessel, can be viewed from the same position when water is poured into the vessel.*

(iv) *A print appears to be raised when a glass slab is placed over it.*

(v) *A piece of paper stuck at the bottom of a glass slab appears to be raised when seen from above.*

(vi) *A tank appears shallow than its actual depth.*

(vii) *A person's legs appear to be short when standing in a tank.*

EXAMPLES

4.13 TRANSMISSION OF LIGHT FROM A DENSER MEDIUM (GLASS OR WATER) TO A RARER MEDIUM (AIR) AT DIFFERENT ANGLES OF INCIDENCE

Consider the refraction of light from a denser medium to a rarer medium. When a light ray travelling in a denser medium falls on the surface separating it from a rarer medium, it is partly reflected back into the denser medium and partly refracted in the rarer medium. The refracted ray bends away from the normal on the surface at the point of incidence obeying the laws of refraction. Now we shall consider this process of reflection and refraction at different angles of incidence.

Case (i) when the angle of incidence is small ($i < C$) : In Fig 4.42, a light ray AO travelling in glass is incident at the glass-air interface at a small angle of incidence i. It is partly reflected and partly refracted. We get a weak reflected ray OC and a strong refracted ray OB. Since the incident ray bends away from the normal when it suffers refraction from glass to

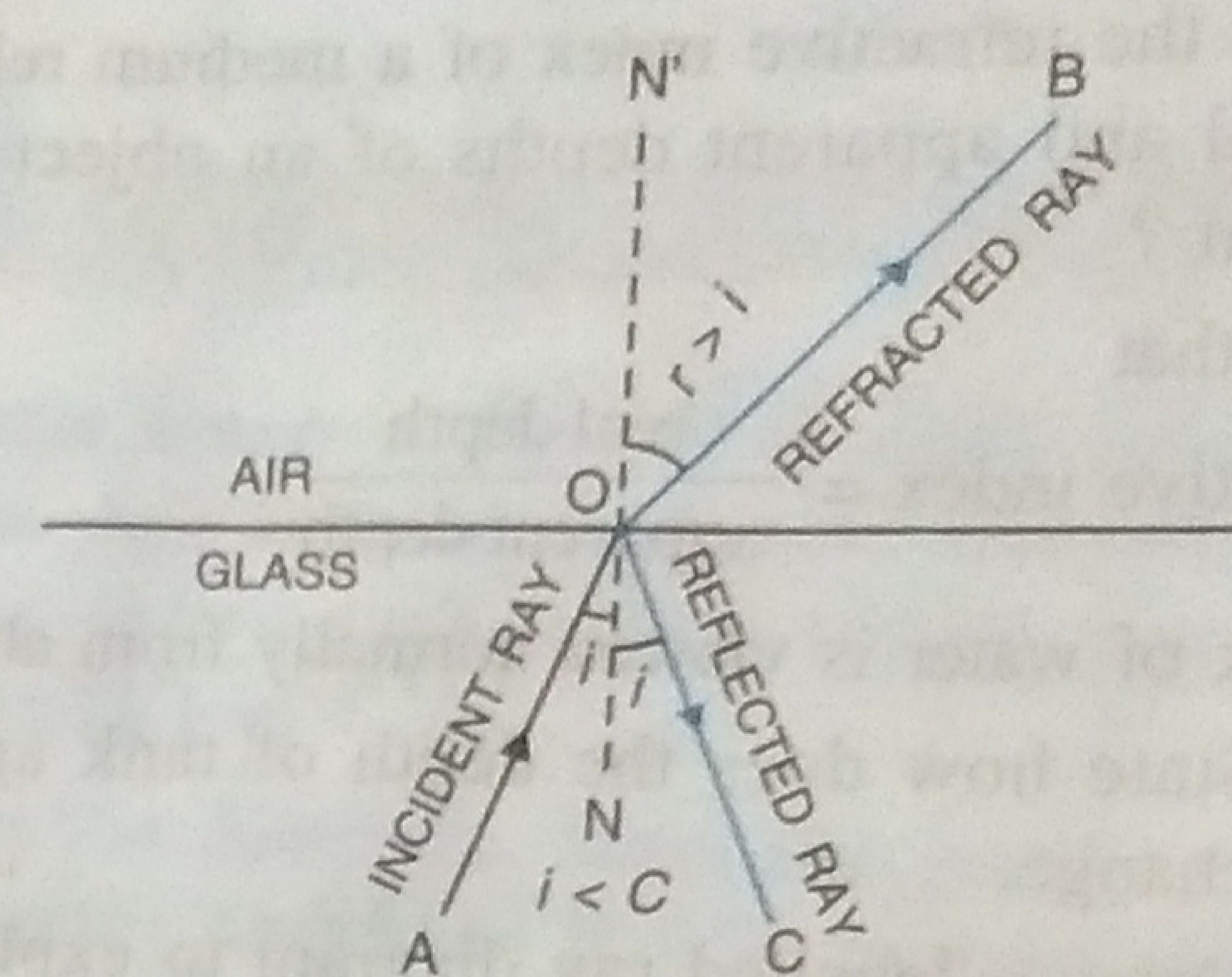

Fig. 4.42 Refraction from glass to air when $i < C$

air, therefore the angle of refraction r is greater than the angle of incidence i.

Now if the angle of incidence i is gradually increased, the angle of refraction r also increases, but the intensity of refracted ray keeps on decreasing. Finally the angle of refraction r reaches its maximum possible value equal to 90° at a certain angle of incidence $i = C$. Here C is called the *critical angle**.

Case (ii) when the angle of incidence is equal to the critical angle ($i = C$) : At angle of incidence equal to the critical angle ($i = C$), the angle of refraction becomes 90° as shown in Fig. 4.43. The refracted ray is along the glass-air interface and it is very weak.

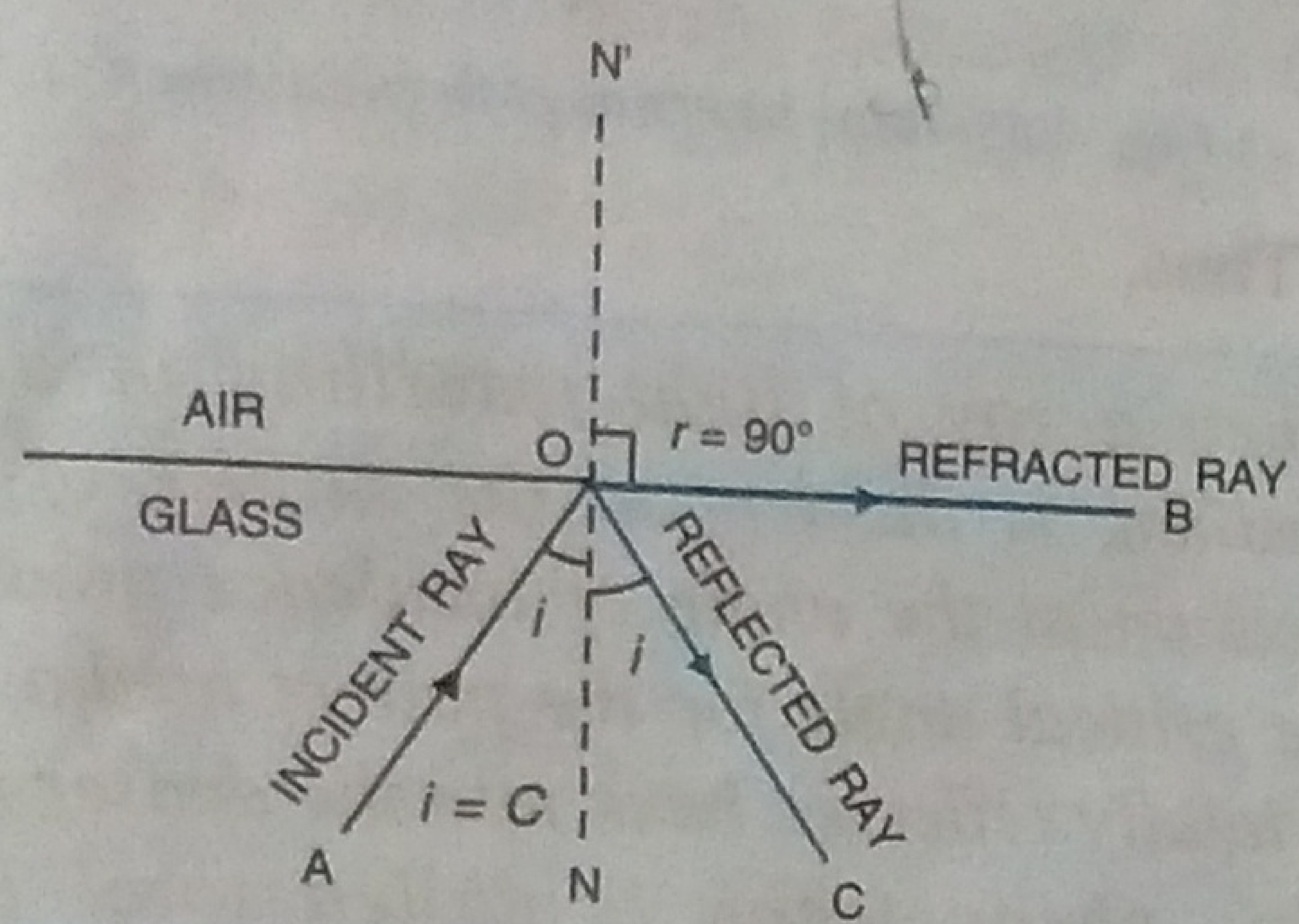

Fig. 4.43 Refraction from glass to air when i = C

In Fig. 4.43, for the incident ray AO at $i = C$, the refracted ray is OB and the reflected ray is OC.

Case (iii) when the angle of incidence is greater than the critical angle ($i > C$) : Fig. 4.44 shows that for the incident ray AO at an angle of incidence i greater than the critical angle C, *no refracted ray is obtained* and the incident ray is totally reflected as OC. Such that $\angle AON = \angle CON$.

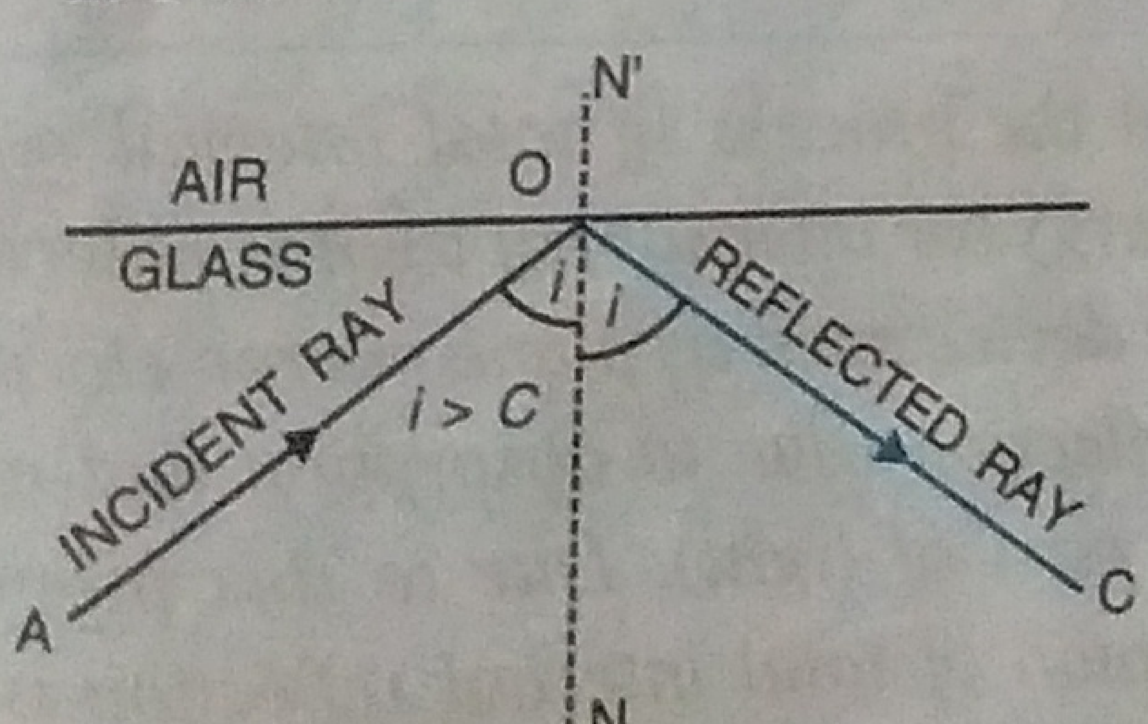

Fig. 4.43 Total reflection when i > C (no refraction)

* Symbol i_c is also used to denote the critical angle.

4.14 CRITICAL ANGLE

We have read that when a ray of light passes from a denser medium to a rarer medium, at a certain angle of incidence $i = C$, the angle of refraction becomes 90°, *i.e.*, at $i = C$, $r = 90°$. The angle C is called the *critical angle*. Thus,

Critical angle is the angle of incidence in the denser medium corresponding to which the angle of refraction in the rarer medium is 90°.

4.15 RELATIONSHIP BETWEEN THE CRITICAL ANGLE AND THE REFRACTIVE INDEX ($\mu = 1/\sin C$)

In Fig. 4.43, AO is an incident ray from glass to air at an angle of incidence $i = C$ (critical angle) for which the angle of refraction r is 90°. Therefore, the refractive index of air with respect to glass is

$$ {}_g\mu_a = \frac{\sin C}{\sin 90°} $$

But $\sin 90° = 1 \quad \therefore \quad {}_g\mu_a = \sin C$

But refractive index of glass with respect to air is

$$ {}_a\mu_g = \frac{1}{{}_g\mu_a} $$

$$ \therefore \quad {}_a\mu_g = \frac{1}{\sin C} = \text{cosec } C \qquad \text{...(4.16)} $$

Thus, knowing the refractive index of the denser medium with respect to the rarer medium, we can calculate the critical angle C for that pair of media.

Examples :

(1) For glass, refractive index ${}_a\mu_g = \frac{3}{2}$

$\therefore \quad \sin C = \frac{1}{{}_a\mu_g} = \frac{2}{3}$

But $\sin 42° = \frac{2}{3}$, $\therefore \; C = 42°$

(2) For water, refractive index ${}_a\mu_w = \frac{4}{3}$

$\therefore \quad \sin C = \frac{1}{{}_a\mu_w} = \frac{3}{4}$

But $\sin 49° = \frac{3}{4}$, $\therefore \; C = 49°$