L. HOSPITAL

1. $\lim_{x\to 0} (\cos x)^{\cot^2 x}$

Let $\lim_{x\to 0} (\cos x)^{\cot^2 x} = y$

$\log y = \lim_{x\to 0} \log (\cos x)^{\cot^2 x}$

$= \lim_{x\to 0} \cot^2 x \log(\cos x)$

$= \lim_{x\to 0} \frac{\log(\cos x)}{\tan^2 x}$ $\left(\frac{0}{0} \text{ form}\right)$ $\left[\cot^2 x = \frac{1}{\tan^2 x}\right]$

$= \lim_{x\to 0} \frac{\frac{d}{dx}\log(\cos x)}{\frac{d}{dx}\tan^2 x}$

$= \lim_{x\to 0} \frac{\frac{1}{\cos x}(-\sin x)}{2\tan x \sec^2 x}$

$= \frac{-1}{2} \lim_{x\to 0} \frac{\tan x}{\tan x \sec^2 x}$

$= \frac{-1}{2} \lim_{x\to 0} \cos^2 x$

$= -\frac{1}{2} \times 1^2$

$\therefore \log_e y = -\frac{1}{2}$

$\Rightarrow e^{-\frac{1}{2}} = y$

$y = \frac{1}{\sqrt{e}}$. Ans

3. $\lim_{x\to 0} (1+\sin x)^{\cot x} = y$

$\log y = \lim_{x\to 0} \cot x \log(1+\sin x)$

$\Rightarrow \log y = \lim_{x\to 0} \cot x \log(1+\sin x)$

$\Rightarrow \log y = \lim_{x\to 0} \frac{\log(1+\sin x)}{\tan x}$ $\left(\frac{0}{0} \text{ form}\right)$

$\Rightarrow \log y = \lim_{x\to 0} \frac{\frac{1}{1+\sin x}\cos x}{\sec^2 x}$

$= \lim_{x\to 0} \frac{\cos x \times \cos^2 x}{(1+\sin x)}$

$= \frac{1\times 1^2}{1+0}$

$\log_e y = 1$

$e^1 = y$ Ans

4. $\lim_{x\to 0} (1+x)^{2/x}$

Let $y = \lim_{x\to 0} (1+x)^{2/x}$

$\log y = \lim_{x\to 0} \log (1+x)^{2/x}$

$= \lim_{x\to 0} \frac{2}{x} \log(1+x)$

$= 2 \lim_{x\to 0} \frac{\log(1+x)}{x}$ $\left(\frac{0}{0} \text{ form}\right)$

$= 2 \lim_{x\to 0} \frac{\frac{d}{dx}\log(1+x)}{\frac{dx}{dx}}$

$$\log y = 2 \lim_{x \to 0} \frac{\frac{1}{1+x}}{1}$$

$$= 2 \times \frac{\frac{1}{1+0}}{1}$$

$$\log_e y = 2$$

$$e^2 = y \quad \text{Ans}$$

5. $$\lim_{x \to \pi/2} (\cos x \log \tan x)$$

$$= \lim_{x \to \pi/2} \frac{\log \tan x}{\sec x} \qquad \left(\frac{\infty}{\infty} \text{ form}\right)$$

$$= \lim_{x \to \pi/2} \frac{\frac{d}{dx} \log \tan x}{\frac{d}{dx} \sec x}$$

$$= \lim_{x \to \pi/2} \frac{\frac{1}{\tan x} \sec^2 x}{\sec x \tan x}$$

$$= \lim_{x \to \pi/2} \frac{\cos x}{\sin x} \times \frac{1}{\cos x} \times \frac{\cos x}{\sin x}$$

$$= \frac{0}{1^2} = 0$$

(6) $\lim_{x\to 0} \left(1+\frac{1}{x^2}\right)^x$

Let $y = \lim_{x\to 0} \left(1+\frac{1}{x^2}\right)^x$

$\log y = \lim_{x\to 0} x \log\left(1+\frac{1}{x^2}\right)$

$= \lim_{x\to 0} \frac{\log\left(1+\frac{1}{x^2}\right)}{1/x}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x\to 0} \frac{\frac{1}{1+1/x^2}\left(0-\frac{2}{x^3}\right)}{-1/x^2}$

$= \lim_{x\to 0} \frac{x^2}{(x^2+1)} \times \left(\frac{-2}{x^3}\right) \Big/ \left(-1/x^2\right)$

$= \lim_{x\to 0} \frac{2}{x(x^2+1)} \times x^2$

$= 2\lim_{x\to 0} \frac{x}{x^2+1}$

$= 2\lim_{x\to 0} \frac{1}{\frac{x^2+1}{x}}$

$= 2\lim_{x\to 0} \frac{1}{x+1/x}$

$= 2\times 0$

$\log y = 0 \Rightarrow e^0 = y$

$\Rightarrow y = 1$ Ans

7. $\lim_{y\to 0}\left(\frac{y-\tan^{-1}y}{y-\sin^{-1}y}\right)$ $\left(\frac{0}{0}\text{ form}\right)$

$$= \lim_{y\to 0} \frac{1-\frac{1}{1+y^2}}{1-\frac{1}{\sqrt{1-y^2}}}$$

$$= \lim_{y\to 0} \frac{\frac{1+y^2-1}{(1+y^2)}}{\frac{\sqrt{1-y^2}-1}{\sqrt{1-y^2}}}$$

$$= \lim_{y\to 0} \frac{y^2}{1+y^2} \times \frac{\sqrt{1-y^2}}{\sqrt{1-y^2}-1}$$

$$= \lim_{y\to 0} \frac{y^2}{1+y^2} \times \frac{\sqrt{1-y^2}\left(\sqrt{1-y^2}+1\right)}{(1-y^2)-1}$$

$$= \lim_{y\to 0} \frac{y^2}{1+y^2} \times \frac{\sqrt{1-y^2}\left(\sqrt{1-y^2}+1\right)}{1-y^2-1}$$

$$= -\lim_{y\to 0} \frac{y^2\left(\sqrt{1-y^2}\right)\left(\sqrt{1-y^2}+1\right)}{(1+y^2)\,y^2}$$

$$= -\frac{\sqrt{1-0}\left(\sqrt{1-0}+1\right)}{1+0}$$

$$= -\frac{1(2)}{1}$$

$$= -2 \quad \text{Ans}$$

(8) $\lim_{x \to \pi/2} \frac{\tan 3x}{\tan x}$

$= \lim_{x \to \pi/2} \frac{\sin 3x \cos x}{\cos 3x \sin x}$

$= \lim_{x \to \pi/2} \frac{2 \sin 3x \cos x}{2 \cos 3x \sin x}$

$= \lim_{x \to \pi/2} \frac{\sin 4x + \sin 2x}{\sin 4x - \sin 2x}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to \pi/2} \frac{4\cos 4x + 2\cos 2x}{4\cos 4x - 2\cos 2x}$

$= \frac{4 \times 1 - 2 \times 1}{4 \times 1 + 2 \times 1}$

$= \frac{2}{6}$

$= \frac{1}{3}$ Ans

** If $y = \int_a^{\phi(x)} f(t)\,dt$ Then $\frac{dy}{dx} = f(t)\Big]_{t=\phi(x)}$

$\lim_{x \to 0} \frac{\left(\int_0^x e^{x^2} dx\right)^2}{\int_0^x e^{ex^2} dx}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

$= \lim_{x \to 0} \frac{\frac{d}{dx}\left(\int_0^x e^{x^2} dx\right)^2}{\frac{d}{dx} \int_0^x e^{ex^2} dx}$

$= \lim_{x \to 0} \frac{\left(2 \int_0^x e^{x^2} dx\right) \cdot e^{x^2}}{e^{2x^2}}$

$$= \lim_{x \to 0} \frac{2\int_0^x e^{x^2} dx}{e^{x^2}} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x \to 0} \frac{2 \frac{d}{dx} \int_0^x e^{x^2} dx}{\frac{d}{dx} e^{x^2}}$$

$$= \lim_{x \to 0} \frac{2 e^{x^2}}{e^{x^2} \cdot 2x}$$

$$= 0 \quad \text{Ans}$$

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(2) $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$

$=\lim_{x\to 0}\left(\frac{\sin^2 x - x^2}{x^2\sin^2 x}\right)$

$=\lim_{x\to 0}\frac{\sin^2 x - x^2}{x^4}\cdot\left(\frac{x^2}{\sin^2 x}\right)$

$=\lim_{x\to 0}\frac{\sin^2 x - x^2}{x^4}\lim_{x\to 0}\frac{x^2}{\sin^2 x}$

$=\lim_{x\to 0}\frac{\sin^2 x - x^2}{x^4}\lim_{x\to 0}\left(\frac{x}{\sin x}\right)^2$

$=\lim_{x\to 0}\frac{\sin^2 x - x^2}{x^4}\times 1^2$

$=\lim_{x\to 0}\frac{\sin^2 x - x^2}{x^4}\left(\frac{0}{0}\text{ form}\right)$

$=\lim_{x\to 0}\frac{2\sin x\cos x - 2x}{4x^3}$

$=\lim_{x\to 0}\frac{\sin 2x - 2x}{4x^3}$

$=\lim_{x\to 0}\frac{2\cos 2x - 2}{12x^2}\left(\frac{0}{0}\text{ form}\right)$

$=\lim_{x\to 0}\frac{-2\times 2\sin 2x}{24x}$

$=-\frac{1}{6}\lim_{x\to 0}\frac{\sin 2x}{x}\left(\frac{0}{0}\text{ form}\right)$

$=-\frac{1}{6}\lim_{x\to 0}\frac{2\cos 2x}{1}=-\frac{1}{3}$ Ans

NTO

$\lim_{x\to 0}\frac{x}{\sin x}$

$=\lim_{x\to 0}\frac{\frac{dx}{dx}}{\frac{d\sin x}{dx}}$

$=\lim_{x\to 0}\frac{1}{\cos x}$

$=\frac{1}{\cos 0}$

$=1$