(1) Verify Rolle's theorem for each of the following

$f(x) = 4\sin x$, in $[0, \pi]$

Clearly $f(x) = 4\sin x$ is continuous in $[0, \pi]$

$$\frac{d}{dx} f(x) = 4\cos x$$

$f(x)$ is derivable in $(0, \pi)$

$$f(0) = 4\sin 0 = 0$$

$$f(\pi) = 4\sin \pi = 0$$

$$f(0) = f(\pi)$$

So f satisfy all conditions of Rolle's Theo

To verify Rolle's Theorem let

$$f'(c) = 0$$

$$4\cos c = 0$$

$$\cos c = 0$$

$$\cos c = \cos \pi/2$$

$$c = \pi/2 \in (0, \pi)$$

Hence Rolle's Theorem is verified

(2) $f(x) = \sin x + \cos x$ in $[0, \pi/2]$, verify Rolle's Theo

$f(0) = \sin 0 + \cos 0 = 1$

$f(\pi/2) = \sin \pi/2 + \cos \pi/2$

$= 1 + 0$

$= 1$

(i) $f(0) = f(\pi/2)$

(ii) clearly $f(x)$ in continuous in $[0, \frac{\pi}{2}]$

(iii) $f'(x) = \frac{d}{dx} f(x)$

$= \cos x - \sin x$

$\because f$ is derivable in $(0, \pi/2)$

$\therefore f$ satisfied all condition of Rolle's Theo

To verify Rolle's Theo,

Let $f'(c) = 0$

$\cos c - \sin c = 0$

$\tan c = 1$

$c = \frac{\pi}{4} \in (0, \pi/2)$

Hence Rolle's Theo is verified

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(3) $f(x) = x(x-2)^2$ in $0 \le x \le 2$
verify Rolle's Theo.

$$f(x) = x(x-2)^2 = x(x^2-4x+4) = x^3-4x^2+4x$$

$f(0) = 0$

$f(2) = 0$

(i) $f(0) = f(2)$

(ii) $f(x) = x(x-2)^2$ is continuous in $[0, 2]$

(iii) $f'(x) = \frac{d}{dx}(x^3-4x^2+4x)$

$= 3x^2-8x+4$

$\therefore$ f is derivable.

$\therefore$ ~~f satisfy~~
all conditions of Rolle's Theo are satisfied

To verify Rolle's Theo;

let $f'(c) = 0$

$\Rightarrow 3c^2-8c+4=0$

$\Rightarrow 3c^2-6c-2c+4=0$

$\Rightarrow 3c(c-2)-2(c-2)=0$

$(c-2)(3c-2)=0$

$c=2$ | $c = 2/3 \in (0, 2)$

Hence Rolle's Theo is verified.

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4) Verify Rolle's Theorem for $f(x) = \log\left[\frac{x^2+ab}{x(a+b)}\right]$

$x \in [a, b]$

(i) $f(x)$ is continuous in $[a, b]$

~~$f(x)$ is derivable in (a,b)~~

(ii) $f(x) = \log(x^2+ab) - \log x - \log(a+b)$

$$f'(x) = \frac{1}{x^2+ab} \cdot 2x - \frac{1}{x}$$

$$= \frac{2x}{x^2+ab} - \frac{1}{x}$$

f is derivable in (a, b)

$$f(a) = \log(a^2+ab) - \log a - \log(a+b)$$

$$= \log \frac{a^2+ab}{a(a+b)}$$

$$= \log 1$$

$$= 0$$

Similarly $f(b) = 0$

$f(a) = f(b)$

~~Then by Rolle's Theo, $\exists$ at least one p~~

To verify Rolles Theo let $f'(c) = 0$

$$\Rightarrow \frac{2c}{c^2+ab} - \frac{1}{c} = 0$$

$$\frac{2c}{c^2+ab} = \frac{1}{c}$$

$$\Rightarrow 2c^2 = c^2+ab$$

$$2c^2 - c^2 = ab$$

$$c^2 = ab$$

$$c = \sqrt{ab} \in (a, b)$$

(7) $f(x) = x^3 + ax^2 + bx, \quad x \in [1,2]$

It is given that Rolle's Theorem holds good for $f(x)$ at $x = \frac{4}{3}$. find a and b

Since $f(x)$ satisfy all condition of Rolle's then,

$\therefore f(1) = f(2)$

$1 + a + b = 8 + 4a + 2b$

$-7 = 3a + b \quad \text{—(1)}$

$f'(x) = 3x^2 + 2ax + b$

$f'(4/3) = \cancel{3} \times \frac{16}{\cancel{9}\,3} + 2a \cdot \frac{4}{3} + b = 0$

$16 + 8a + 3b = 0$

$8a + 3b = -16 \quad \text{—(2)}$

$8a + 3b = -16 \quad | \times 1$

$3a + b = -7 \quad | \times 3$

$8a + \cancel{3b} = -16$

$9a + \cancel{3b} = -21$

$(-) \quad (-) \quad (+)$

$-a = 5$

$a = -5$

Again

$b = -7 - 3a$

$= -7 - 3 \times (-5)$

$= -7 + 15$

$= 8$