(1) Let R be The set of real numbers let $f: R \to R$ is defined by $f(x) = \cos x$ and $g: R \to R$ is defined by $g(x) = x^3$ prove

$$g \circ f \neq f \circ g$$

Ans $f(x) = \cos x$

$g(x) = x^3$

$f \circ g(x)$	$g \circ f(x)$
$= f \circ (x^3)$	$g \circ \cos x$
$= f(x^3)$	$g(\cos x)$
$= \cos x^3$	$= (\cos x)^3$

$$\cos x^3 \neq (\cos x)^3$$

$$g \circ f \neq f \circ g$$

Scanned with CamScanner

(2) Find $f \circ g$ and $g \circ f$ if $f: R \to R$ and $g: R \to R$ is defined by $f(x) = 8x^3$ is defined $g(x) = x^{1/3}$

$f(x) = x^3$
$g(x) = x^{1/3}$

Now $f \circ g(x)$
$f(x^{1/3})$
$= 8(x^{1/3})^3$
$= 8x$

$\therefore f \circ g(x) = 8x$

$g \circ f(x)$
$= g(8x^3)$
$= (8x^3)^{1/3}$
$= (2x)^{3 \cdot 1/3}$
$= 2x$

$g \circ f(x) = 2x$

(3) $f: R \to R$ defined by $f(x) = 2x - 3$, $g: R \to R$, $g(x) = \frac{x+3}{2}$, Show $f \circ g = I_R = g \circ f$

$f \circ g(x)$
$= f\left(\frac{x+3}{2}\right)$
$= 2\left(\frac{x+3}{2}\right) - 3$
$= x + \cancel{3} - \cancel{3}$
$= x$

$\therefore f \circ g = I_R$

$g \circ f(x)$
$= g(2x - 3)$
$= \frac{1}{2}(2x - \cancel{3} + \cancel{3})$
$= x$

$g \circ f = I_R$.

(4) Find the inverse of function of $f(x) = \frac{x-1}{x+1}$, and verify that $f \circ f^{-1}$ is an identity function.

$$f(x) = \frac{x-1}{x+1}$$

$$y = \frac{x-1}{x+1}$$

$$yx + y = x - 1$$

$$y + 1 = x - yx$$

$$y + 1 = x(1-y)$$

$$x = \frac{y+1}{1-y}$$

$$f^{-1}(y) = \frac{y+1}{1-y}$$

Now $f \circ f^{-1}(y)$

$$= f\left(\frac{y+1}{1-y}\right)$$

$$= \frac{\frac{y+1}{1-y} - 1}{\frac{y+1}{1-y} + 1}$$

$$= \frac{2y}{2} = y$$

$\therefore f \circ f^{-1}(y) = y$

$\therefore f \circ f^{-1}$ is identity function.

(5) Show that $f: R \to R$ defined by $f(x) = \frac{x}{x^2+1}$ for all $x \in R$ is neither one-one nor onto

If possible let $f(x_1) = f(x_2)$

$$\frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$$

$$\Rightarrow x_1 x_2^2 + x_1 = x_1^2 x_2 + x_2$$

$$\Rightarrow x_1 x_2 (x_2 - x_1) = (x_2 - x_1)$$

$$x_1 x_2 (x_2 - x_1) - (x_2 - x_1) = 0$$

$$(x_2 - x_1)(x_1 x_2 - 1) = 0$$

either $x_2 = x_1$ | or $x_1 x_2 = 1$

Let $x_1 = 2$

$$f(2) = \frac{2}{2^2+1} = \frac{2}{5}$$

$$f(1/2) = \frac{1/2}{(1/2)^2+1} = \frac{1}{2} \times \frac{4}{(4+1)} = \frac{2}{5}$$

$\therefore f(2) = f(1/2)$

$\therefore f$ is not one-one

(4)

Again $f(x) = y$

$$\frac{x}{x^2+1} = y$$

$$x = x^2y + y$$

$$x^2y - x + y = 0$$

$$x = \frac{1 \pm \sqrt{1 - 4 \times y \times y}}{2y}$$

$$x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

x is ~~img~~ imaginary if

$1-4y^2 < 0$

Let $y = 1$

$\therefore x = \dfrac{1 \pm \sqrt{1-4\times1^2}}{2\times1}$

$= \dfrac{1 \pm \sqrt{3}\, i}{2}$

$\therefore y = 1$ ~~the~~ $\in$ co-domain set has no- preimage

$\therefore$ f is not onto