(1) Find The domain of function

$$f(x) = \sqrt{x-1} + \sqrt{6-x}$$

function $f(x)$ is defined if

$x-1 \geq 0$ and $6-x \geq 0$

$x \geq 1$ | $6 \geq x$

$1 \leq x$ | $x \leq 6$

$1 \leq x \leq 6$

domain $= \{x \mid x \in R, 1 \leq x \leq 6\}$

Find range of

$$g(x) = \sin x + \cos x$$

Let $y = (\sin x + \cos x)$

$$y = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)$$

$$= \sqrt{2}\left(\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}\right)$$

$$= \sqrt{2}\sin(x - \pi/4)$$

Now $-1 \leq \sin(x-\pi/4) \leq 1$

$$-\sqrt{2} \leq \sqrt{2}\sin\left(x-\frac{\pi}{4}\right) \leq \sqrt{2}$$

~~Range = {y | y ∈ R,~~

Range $= \{y \mid -\sqrt{2} \leq y \leq \sqrt{2}, y \in R\}$

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3. Find domain of $f(x) = \sqrt{15+x-2x^2}$

$15+x-2x^2$

$= 15+6x-5x-2x^2$

$= 15-5x+6x-2x^2$

$= 5(3-x)+2x(3-x)$

$= (3-x)(5+2x)$

$f(x)$ is defined if ~~$(3-x)\geq 0$~~

$(3-x)(5+2x) \geq 0$ — (1)

ie either $3-x\geq 0$ and $5+2x\geq 0$

or $(3-x)\leq 0$ and $5+2x\leq 0$ — (2)

From (1) $3-x\geq 0$ and $5+2x\geq 0$

$3\geq x$	$2x\geq -5$
$x\leq 3$	$x\geq -5/2$
	$-2.5\leq x$

$-2.5\leq x\leq 3$

From (2)

$3-x\leq 0$	$5+2x\leq 0$
$3\leq x$	$2x\leq -5$
	$x\leq -5/2$

There is no common region

$\therefore$ function is defined in $-2.3 \le x \le 3$.

if

(4) Find the domain of each of the following

$\sin^{-1}\left(\log_2 x^2/2\right)$

~~logarithm is defined if $\frac{x^2}{2} > 0$~~

function is defined if

$$-1 \le \log_2 (x^2/2) \le 1$$

$$2^{-1} \le \frac{x^2}{2} \le 2^1$$

$$\frac{1}{2} \le \frac{x^2}{2} \le 2$$

$$1 \le x^2 \le 4$$

$$1 \le x \le 2$$

or $-2 \le x \le -1$

Ans

(5) $f(x) = \frac{x}{1+x^2}$, $x \in R$

$$y = \frac{x}{1+x^2}$$

$$(1+x^2)y = x$$

$$x^2y - x + y = 0$$

$$x = \frac{+1 \pm \sqrt{1^2 - 4 \times 1 \times y^2}}{2 \times y}$$

x is real if $1 - 4y^2 \ge 0$

$$1 \ge 4y^2$$

$$y^2 \le \frac{1}{4}$$

$$-\frac{1}{2} \le y \le 1/2$$

Range ~~of~~ $\{y \mid y \in R$ and $-\frac{1}{2} \le y \le 1/2\}$

⑥ $f(x) = 3x^2 - 6x + 4$, for what values of x is $3f(x) = f(3x)$

Ans

$$3(3x^2 - 6x + 4) = 3(3x)^2 - 6(3x) + 4$$

$$9x^2 - \cancel{18x} + 12 = 27x^2 - \cancel{18x} + 4$$

~~$\Rightarrow 3x^2 - 6x + 4 = 9x^2 - 6x + 4$~~

$$18x^2 - 8 = 0$$

~~$6(3x^2 - 1) = 0$~~ $\quad 2(9x^2 - 4) = 0$

~~$3x^2 = 1$~~ $\quad 9x^2 - 4 = 0$

~~$x^2 = +1/3$~~ $\quad 9x^2 = 4$

~~$x = \pm\frac{1}{\sqrt{3}}$~~ $\quad x^2 = \frac{4}{9}$

$$x = \pm\frac{2}{3}$$

⑦ $y = g(x) = \dfrac{ax+b}{cx-a}$, Then prove $g(y) = x$

$$g(y) = \frac{ay+b}{cy-a}$$

$$= \frac{a\left(\frac{ax+b}{cx-a}\right) + b}{c\left(\frac{ax+b}{cx-a}\right) - a}$$

$$= \frac{a^2x + \cancel{ab} + bcx - \cancel{ab}}{\cancel{acx} + bc - \cancel{acx} + a^2}$$

$$= \frac{x\,\cancel{(a^2+bc)}}{\cancel{(a^2+bc)}} = x$$

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(8) If $f(x) = (a-x^n)^{1/n}$ where $a>0$ and n is positive integer show that $f[f(x)] = x$

$$f(x) = (a-x^n)^{1/n} \Rightarrow f^n(x) = (a-x^n)$$

$$f(f(x)) = \{a-(f(x))^n\}^{1/n}$$

$$= \{a-(a-x^n)\}^{1/n}$$

$$f(f(x)) = (x^n)^{1/n}$$

$$f(f(x)) = x$$

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(9) $f(x) = \frac{4^x}{4^x+2}$ prove that $f(x)+f(1-x) = 1$ and hence $f\left(\frac{1}{1997}\right)+f\left(\frac{2}{1997}\right)+\cdots+f\left(\frac{1996}{1997}\right) = 998$

$$f(x) = \frac{4^x}{4^x+2}$$

$$f(1-x) = \frac{4^{1-x}}{4^{1-x}+2} = \frac{\frac{4}{4^x}}{\frac{4}{4^x}+2}$$

$$= \frac{4/\cancel{4^x}}{\frac{4+2\times 4^x}{\cancel{4^x}}}$$

$$= \frac{2}{2+4^x}$$

Now $f(x)+f(1-x)$

$$= \frac{\cancel{2+4^x}}{\cancel{2+4^x}} = 1$$

$\therefore \cancel{f(x) + f(1-x) = }$

$$\therefore f(x) + f(1-x) = 1$$

$$f\left(\frac{1}{1997}\right) + f\left(\frac{2}{1997}\right) + f\left(\frac{3}{1997}\right) + \cdots + f\left(\frac{1996}{1997}\right)$$

$$= \left\{ f\left(\frac{1}{1997}\right) + f\left(\frac{1996}{1997}\right) \right\} + \cdots + 998 \text{ (group)}$$

$$= 1 + 1 + \cdots + 1 \quad (998 \text{ times})$$

$$= 998$$

(10) Find the domain of each of the following functions

$\log_e \frac{2+x}{2-x}$

The function is defined if

$$\frac{2+x}{2-x} > 0$$

$\cancel{2+x > 0}$ it is possible

$\cancel{x > -2}$ if $2+x > 0$ and $2-x > 0$

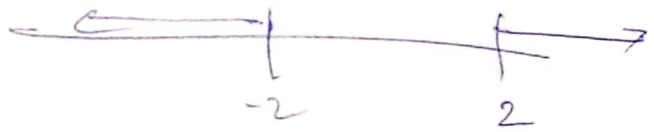
$x > -2$ | $2 > x$

$-2 < x$ | $x < 2$

$\therefore -2 < x < 2$

or $2+x < 0$ and $2-x < 0$

$x < -2$ and $2 < x$



domain $-2 < x < 2$