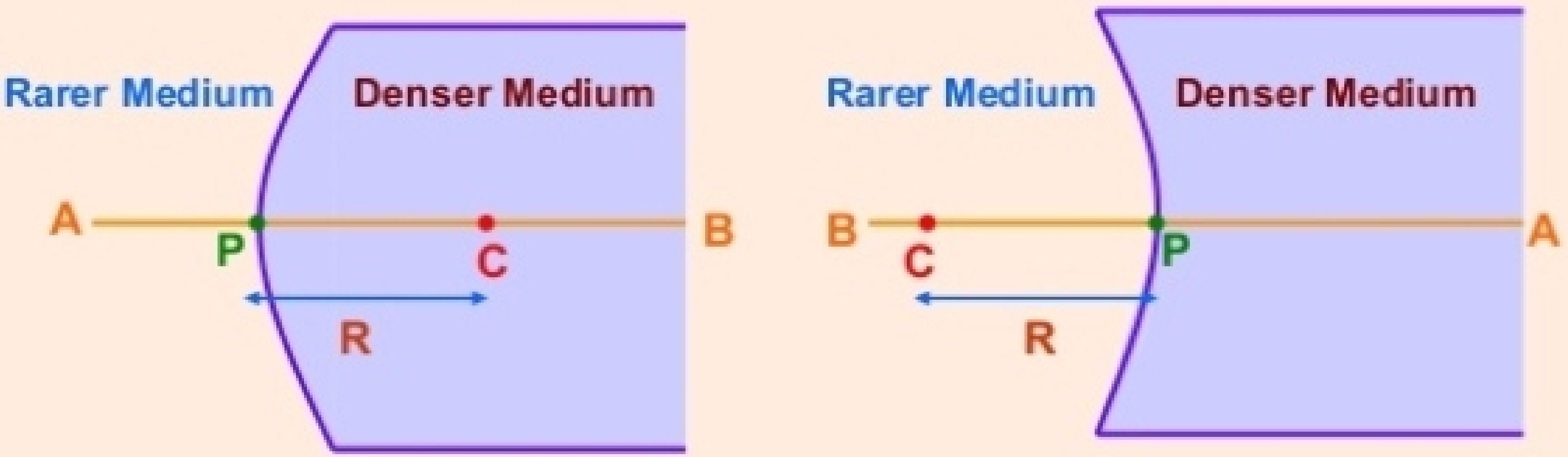
Spherical Refracting Surfaces:

A spherical refracting surface is a part of a sphere of refracting material.

A refracting surface which is convex towards the rarer medium is called convex refracting surface.

A refracting surface which is concave towards the rarer medium is called concave refracting surface.



APCB – Principal Axis
C – Centre of Curvature
P – Pole
R – Radius of Curvature

Assumptions:

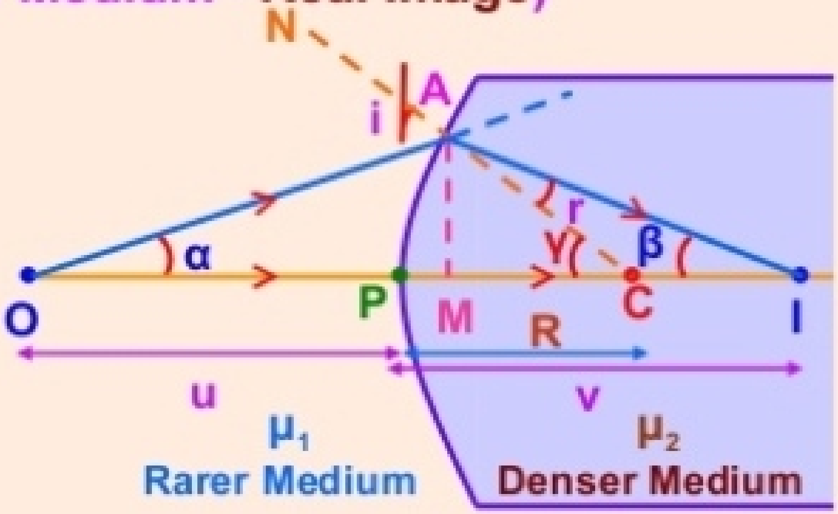
1. Object is the point object lying on the principal axis.
2. The incident and the refracted rays make small angles with the principal axis.
3. The aperture (diameter of the curved surface) is small.

New Cartesian Sign Conventions:

1. The incident ray is taken from left to right.
2. All the distances are measured from the pole of the refracting surface.
3. The distances measured along the direction of the incident ray are taken positive and against the incident ray are taken negative.
4. The vertical distances measured from principal axis in the upward direction are taken positive and in the downward direction are taken negative.

Refraction at Convex Surface:

(From Rarer Medium to Denser Medium - Real Image)



$i = \alpha + \gamma$

$\gamma = r + \beta$ or $r = \gamma - \beta$

$\tan \alpha = \frac{MA}{MO}$ or $\alpha = \frac{MA}{MO}$

$\tan \beta = \frac{MA}{MI}$ or $\beta = \frac{MA}{MI}$

$\tan \gamma = \frac{MA}{MC}$ or $\gamma = \frac{MA}{MC}$

According to Snell's law,

$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$ or $\frac{i}{r} = \frac{\mu_2}{\mu_1}$ or $\boxed{\mu_1 i = \mu_2 r}$

Substituting for i, r, α, β and γ, replacing M by P and rearranging,

$$\frac{\mu_1}{PO} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

Applying sign conventions with values,

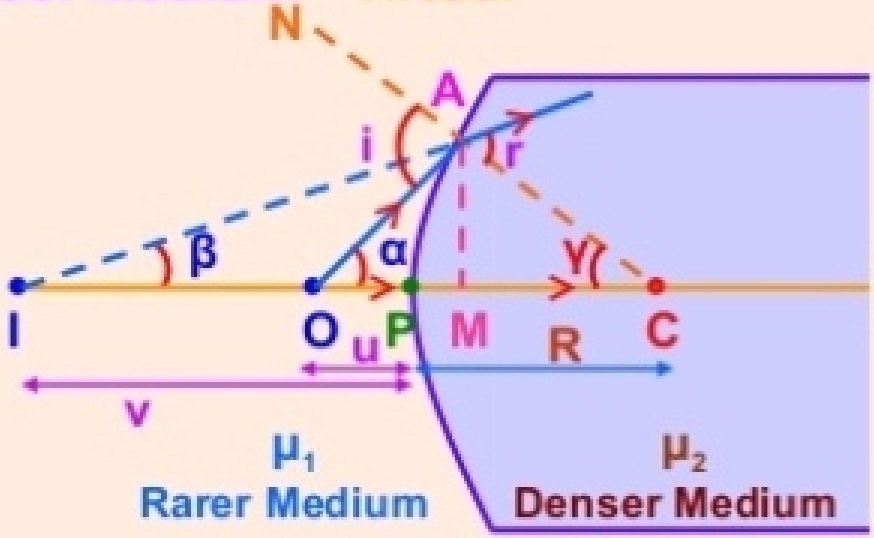
PO = - u, PI = + v and PC = + R

$$\boxed{\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$$

Refraction at Convex Surface:

(From Rarer Medium to Denser Medium - Virtual Image)

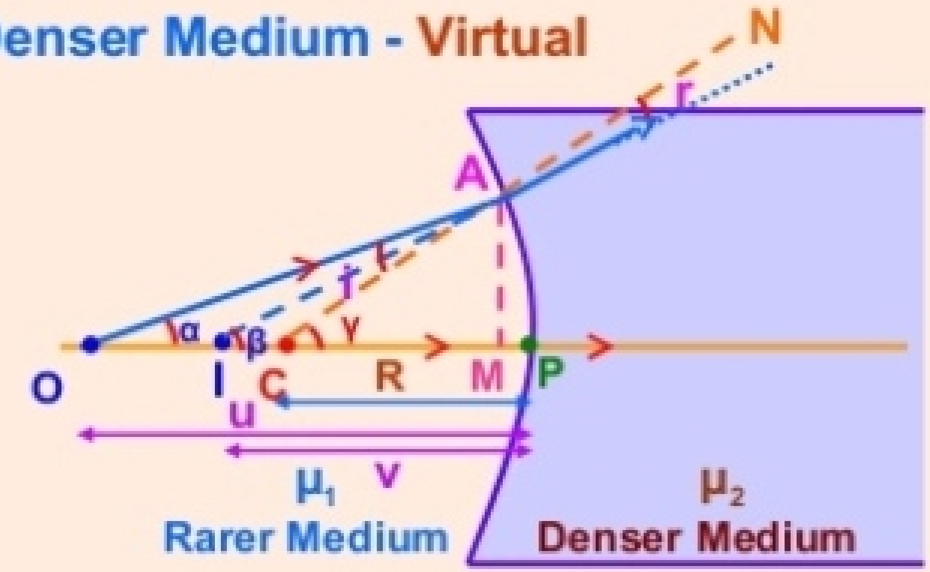
$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$



Refraction at Concave Surface:

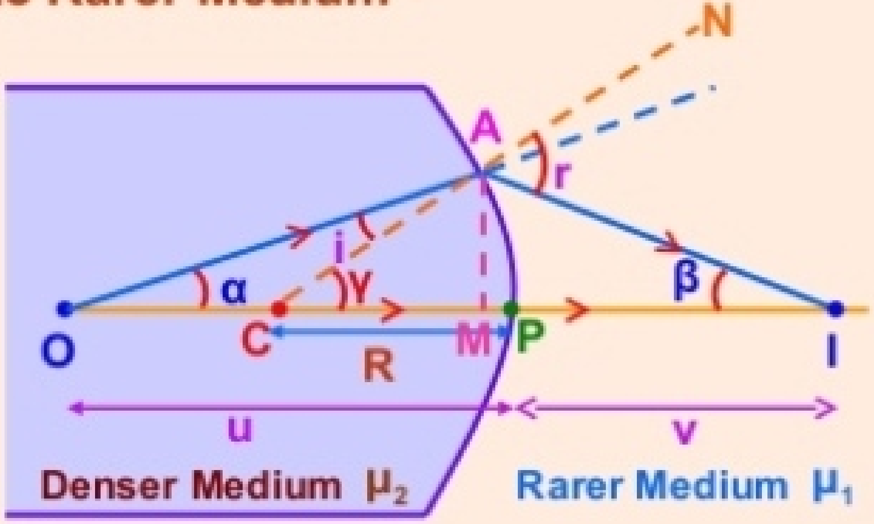
(From Rarer Medium to Denser Medium - Virtual Image)

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$



Refraction at Convex Surface:

(From Denser Medium to Rarer Medium - Real Image)



$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Refraction at Convex Surface:

(From Denser Medium to Rarer Medium - Virtual Image)

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Refraction at Concave Surface:

(From Denser Medium to Rarer Medium - Virtual Image)

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Note:

1. Expression for 'object in rarer medium' is same for whether it is real or virtual image or convex or concave surface.

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

2. Expression for 'object in denser medium' is same for whether it is real or virtual image or convex or concave surface.

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

3. However the values of u, v, R, etc. must be taken with proper sign conventions while solving the numerical problems.

4. The refractive indices μ_1 and μ_2 get interchanged in the expressions.

Lens Maker's Formula:

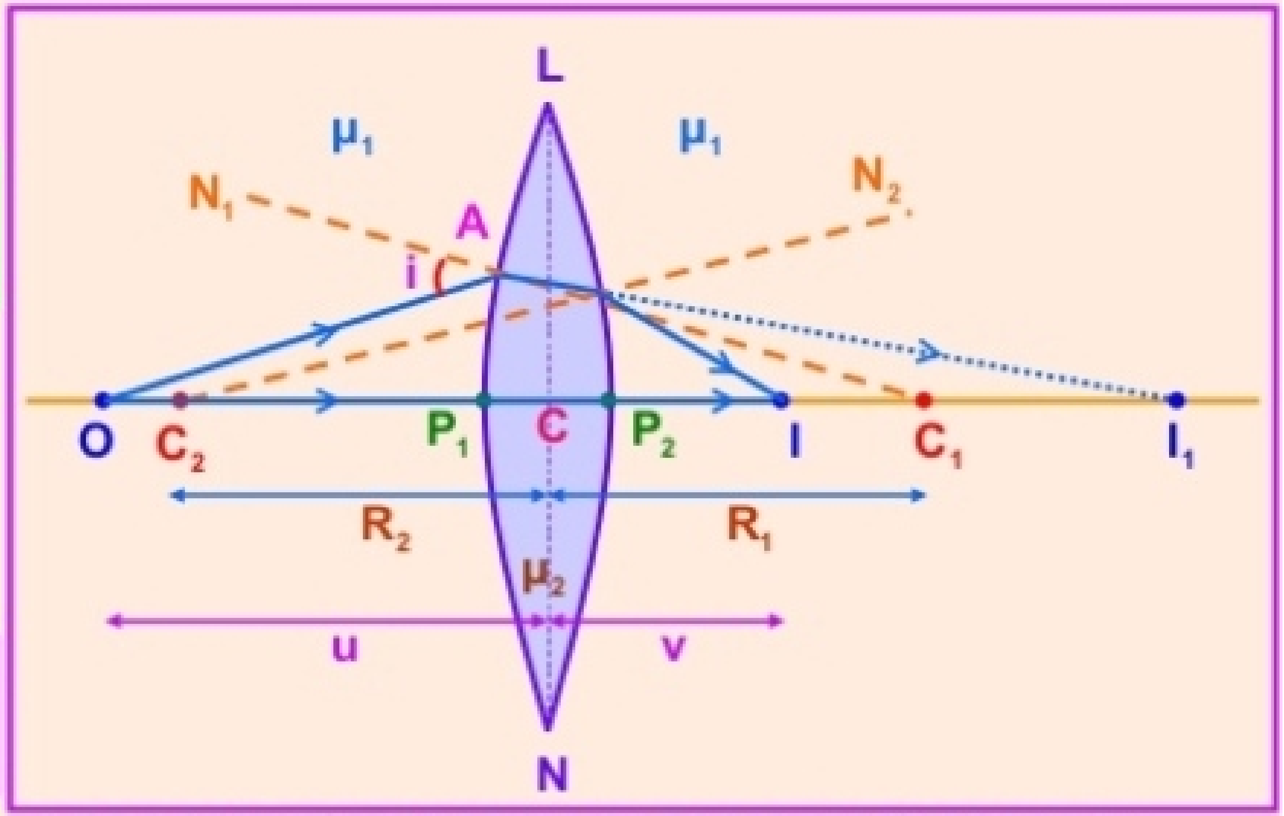
For refraction at LP_1N,

$$\frac{\mu_1}{CO} + \frac{\mu_2}{CI_1} = \frac{\mu_2 - \mu_1}{CC_1}$$

(as if the image is formed in the denser medium)

For refraction at LP_2N,

$$\frac{\mu_2}{-CI_1} + \frac{\mu_1}{CI} = \frac{-(\mu_1 - \mu_2)}{CC_2}$$



(as if the object is in the denser medium and the image is formed in the rarer medium)

Combining the refractions at both the surfaces,

$$\frac{\mu_1}{CO} + \frac{\mu_1}{CI} = (\mu_2 - \mu_1)\left(\frac{1}{CC_1} + \frac{1}{CC_2}\right)$$

Substituting the values with sign conventions,

$$\frac{1}{-u} + \frac{1}{v} = \frac{(\mu_2 - \mu_1)}{\mu_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Since $\mu_2 / \mu_1 = \mu$

$$\frac{1}{-u} + \frac{1}{v} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or

$$\frac{1}{-u} + \frac{1}{v} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

When the object is kept at infinity, the image is formed at the principal focus.

i.e. $u = -\infty$, $v = +f$.

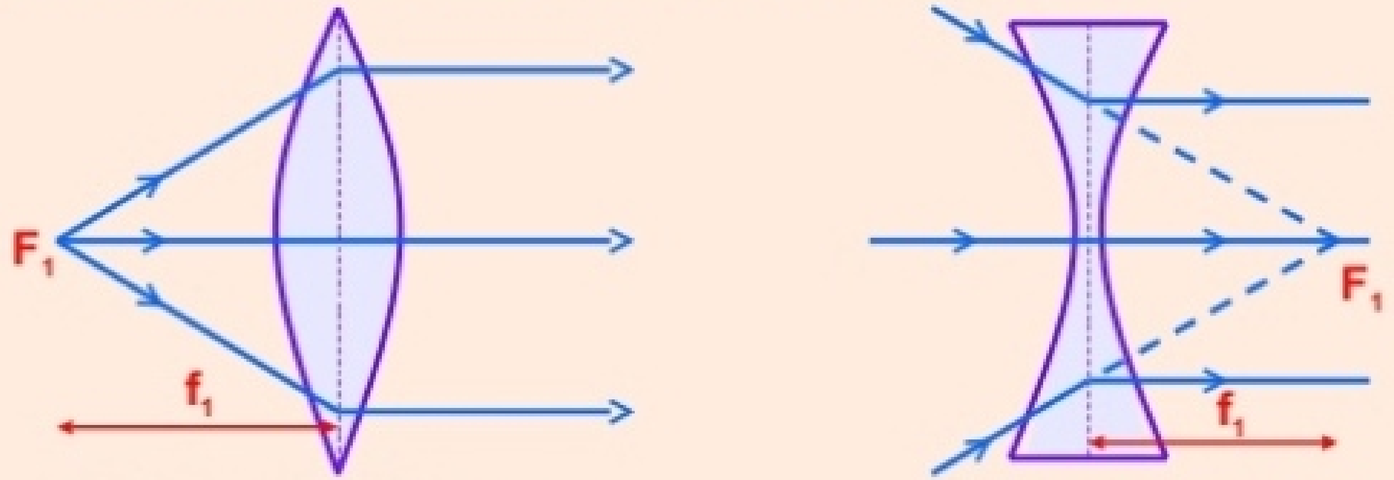
So, $$\boxed{\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

This equation is called 'Lens Maker's Formula'.

Also, from the above equations we get, $$\boxed{\frac{1}{-u} + \frac{1}{v} = \frac{1}{f}}$$

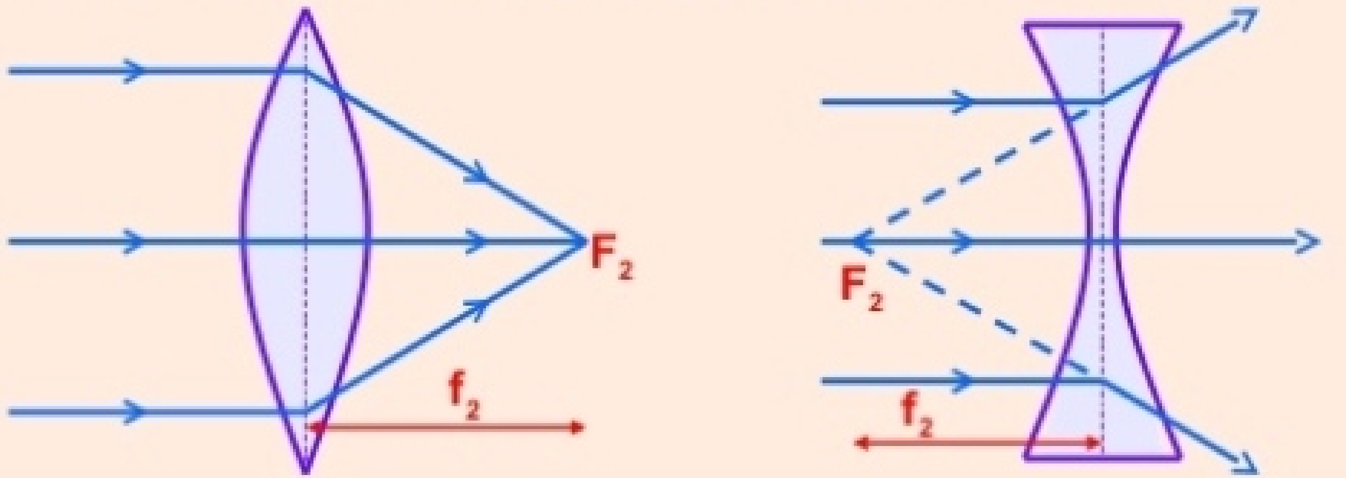
First Principal Focus:

First Principal Focus is the point on the principal axis of the lens at which if an object is placed, the image would be formed at infinity.



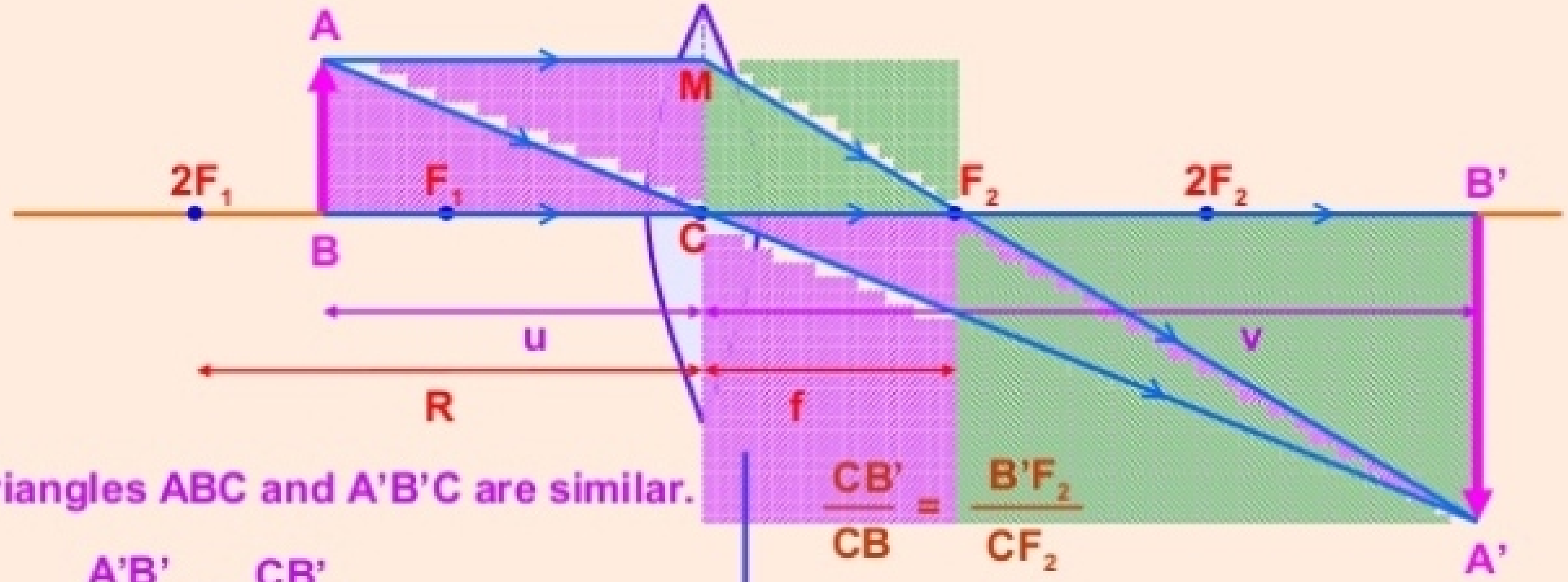
Second Principal Focus:

Second Principal Focus is the point on the principal axis of the lens at which the image is formed when the object is kept at infinity.



Thin Lens Formula (Gaussian Form of Lens Equation):

For Convex Lens:



Triangles ABC and A'B'C are similar.

$$\frac{A'B'}{AB} = \frac{CB'}{CB}$$

Triangles MCF_2 and $A'B'F_2$ are similar.

$$\frac{A'B'}{MC} = \frac{B'F_2}{CF_2}$$

or $$\frac{A'B'}{AB} = \frac{B'F_2}{CF_2}$$

$$\frac{CB'}{CB} = \frac{B'F_2}{CF_2}$$

$$\frac{CB'}{CB} = \frac{CB' - CF_2}{CF_2}$$

According to new Cartesian sign conventions,

$CB = -u$, $CB' = +v$ and $CF_2 = +f$.

$$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Linear Magnification:

Linear magnification produced by a lens is defined as the ratio of the size of the image to the size of the object.

$$m = \frac{I}{O}$$

$$\frac{A'B'}{AB} = \frac{CB'}{CB}$$

According to new Cartesian sign conventions,

A'B' = + I, AB = - O, CB' = + v and CB = - u.

$$\frac{+I}{-O} = \frac{+v}{-u} \quad \text{or} \quad m = \frac{I}{O} = \frac{v}{u}$$

Magnification in terms of v and f:

$$m = \frac{f - v}{f}$$

Magnification in terms of v and f:

$$m = \frac{f}{f - u}$$

Power of a Lens:

Power of a lens is its ability to bend a ray of light falling on it and is reciprocal of its focal length. When f is in metre, power is measured in Dioptre (D).

$$P = \frac{1}{f}$$

Combination of thin lenses in contact

Let us consider two lenses A and B of focal length f_1 and f_2 placed in contact with each other. An object is placed at O beyond the focus of the first lens A on the common principal axis.

The lens A produces an image at I_1 . This image I_1 acts as the object for the second lens B. The final image is produced at I as shown in Fig. Since the lenses are thin, a common optical centre P is chosen.

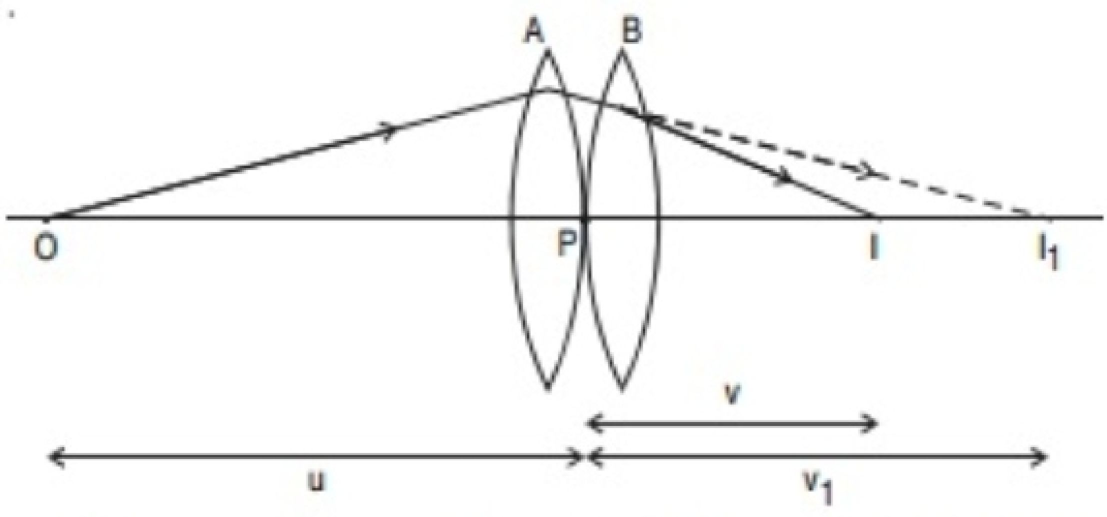


Fig. Image formation by two thin lenses

Let $PO = u$, object distance for the first lens (A), $PI = v$, final image distance and $PI_1 = v_1$, image distance for the first lens (A) and also object distance for second lens (B).

For the image I_1 produced by the first lens A,

$1/v_1$? $1/u = 1/f_1$????..(1)

For the final image I, produced by the second lens B,

$1/v - 1/v_1 = 1/f_2$????..(2)

Adding equations (1) and (2),

$1/v$? $1/u = 1/f_1 + 1/f_2$????..(3)

If the combination is replaced by a single lens of focal length F such that it forms the image of O at the same position I, then

$1/v - 1/u = 1/F$????(4)

From equations (3) and (4)

$1/F = 1/f_1 + 1/f_2$?????(5)

This F is the focal length of the equivalent lens for the combination. The derivation can be extended for several thin lenses of focal lengths f_1, f_2, f_3 ... in contact. The effective focal length of the combination is given by

$1/F = 1/f_1 + 1/f_2 + 1/f_3 +$?????.. ?..(6)

In terms of power, equation (6) can be written as

$P = P_1 + P_2 + P_3 + ...$ ...(7)

Equation (7) may be stated as follows :

The power of a combination of lenses in contact is the algebraic sum of the powers of individual lenses.

The combination of lenses is generally used in the design of objectives of microscopes, cameras, telescopes and other optical instruments.