Relation-

Show that the relation R in the set A $A = \{1,2,3\}$ given by $\{(1,1),(2,2),(3,3),(1,2),(2,3)\}$ is symmetric but neither reflexive nor transitive.

(i) $(1,1),(2,2),(3,3)$ are reflexive

(ii) $(1,2) \in R$ but $(2,1) \notin R$

$\therefore$ R is not symmetric

$(1R2), (2R3)$ but $(1 \not R 3)$

$\therefore$ R is not transitive.

Let S be the set of all real numbers. Show that $R = \{(a,b) : a^2+b^2=1\}$ is neither reflexive nor symmetric but not ~~symmetric~~ transitive

Sol) Let $a=1, b=0$

$a^2+b^2 = 1^2+0 = 1$

$(1,0) \in R$ i.e $1 R 0$

Similarly if $a=0, b=1$

$a^2+b^2 = 0+1^2 = 1$

$\therefore (0,1) \in R$ i.e $0 R 1$

$(1,0) \in R \Rightarrow (0,1) \in R$

$\therefore$ R is symmetric

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$1 R 0$ and $0 R 1$ but $(1,1) \notin R$

$\therefore$ R is not transitive

Let a relation R on $\mathbb{R}$ be defined as $R = \{(a,b) / 1+ab > 0 \quad a,b \in \mathbb{R}\}$
Show that R is reflexive, symmetric but not transitive.

Given $\mathbb{R}$ = set of all real numbers

and $R = \{(a,b) : 1+ab > 0, \; a,b \in \mathbb{R}\}$

Ans

Reflexive

Since $1+a^2 > 0$ is true

$\therefore \; (a,a) \in R$

$\therefore$ R is reflexive

Symmetric

Let $(a,b) \in R$
where $a,b \in \mathbb{R}$

$(a,b) \in R$

$\Rightarrow 1+ab > 0$

$\Rightarrow 1+ba > 0$

$\Rightarrow (b,a) \in R$

R is symmetric

Transitive Let $a = 1$, $b = \frac{1}{3}$, $c = -2$

$1 + 1 \times \frac{1}{3} = \frac{4}{3} > 0$

$\therefore (1, \frac{1}{3}) \in R$

Now $b \cdot c = \frac{1}{3} \times (-2) = -\frac{2}{3}$

$1 + b \cdot c = 1 - \frac{2}{3} = \frac{1}{3} > 0 \Rightarrow (\frac{1}{3}, -2) \in R$

Now $a \cdot c$

$$= 1 \times (-2)$$

$$= -2 < 0$$

$\therefore (a, c) \notin R$

$\therefore (a, b) \in R, (b, c) \in R$

$\Rightarrow (a, c) \notin R$.

$\therefore$ R is not transitive

Relation - 1

1 Determine whether the relation R defined on Z as, $R = \{(x,y) : x-y$ is an integer$\}$ is reflexive symmetric and transitive.

$a \in Z, \quad b \in Z$

$\Rightarrow a-b = \text{Integer} = K_1 \in Z$

$\therefore aRb$

where $K_1 \in Z$

$a-b = \text{integer} = K_1 \Rightarrow (b-a) = -K_1$,

$\therefore aRb \Rightarrow bRa \quad \therefore R$ is symmetric

$a-a=0 = \text{integer} \Rightarrow aRa$

$\therefore R$ is reflexive.

Again

$a-b = K_1$ and let $c \in Z$

$\therefore b-c = K_3 = \text{integer}$

ie bRc

Now $a-\cancel{b}+\cancel{b}-c = K_1+K_2$

$a-c = K_1+K_2 = \text{integer}$

$\therefore aRb, \; bRc \Rightarrow aRc$

$\therefore R$ is transitive.

$\therefore R$ is an equivalence relation.

(2) Let $A = \{x \in Z : 0 \le x \le 12\}$, show that $R = \{(a,b) : a, b \in A, |a-b|$ is divisible by $4\}$ is an equivalence relation.

Let $a, b, c \in A$

and $|a-b| = 4k_1$ = divisible by 4
$k_1 \in Z$.

$\Rightarrow |b-a| = 4k_1$ = divisible by 4

$\therefore aRb \Rightarrow bRa$

$\therefore$ R is symmetric

$a-a = 0$ = divisible by 4

$\therefore aRa$

$\therefore$ R is reflexive

Let $|a-b| = 4k_1 \Rightarrow a-b = \pm 4k_1$

$|b-c| = 4k_2 \Rightarrow b-c = \pm 4k_2$

Now $a-b = \pm 4k_1$

$b-c = \pm 4k_2$

$a - \cancel{b} + \cancel{b} - c = \pm 4k_1 \pm 4k_2$

$a-c = 4(\pm k_1 \pm k_2)$

aRb and $bRc \Rightarrow aRc$

$\therefore$ R is transitive.

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