(C) SIGN CONVENTION AND LENS FORMULA

5.10 SIGN CONVENTION OF MEASUREMENT OF DISTANCES

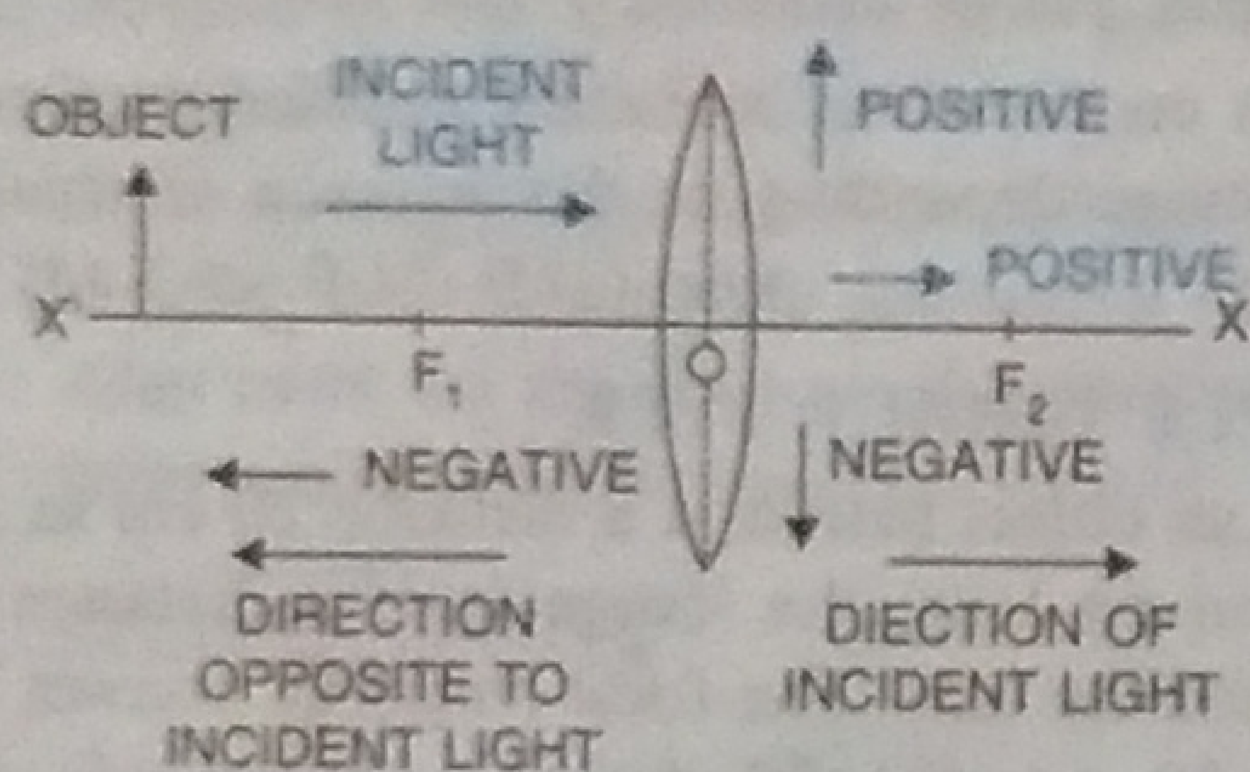
We follow the cartesian sign convention to measure the distances in a lens according to which:

(1) The optical centre of the lens is chosen as the origin of the coordinate system.

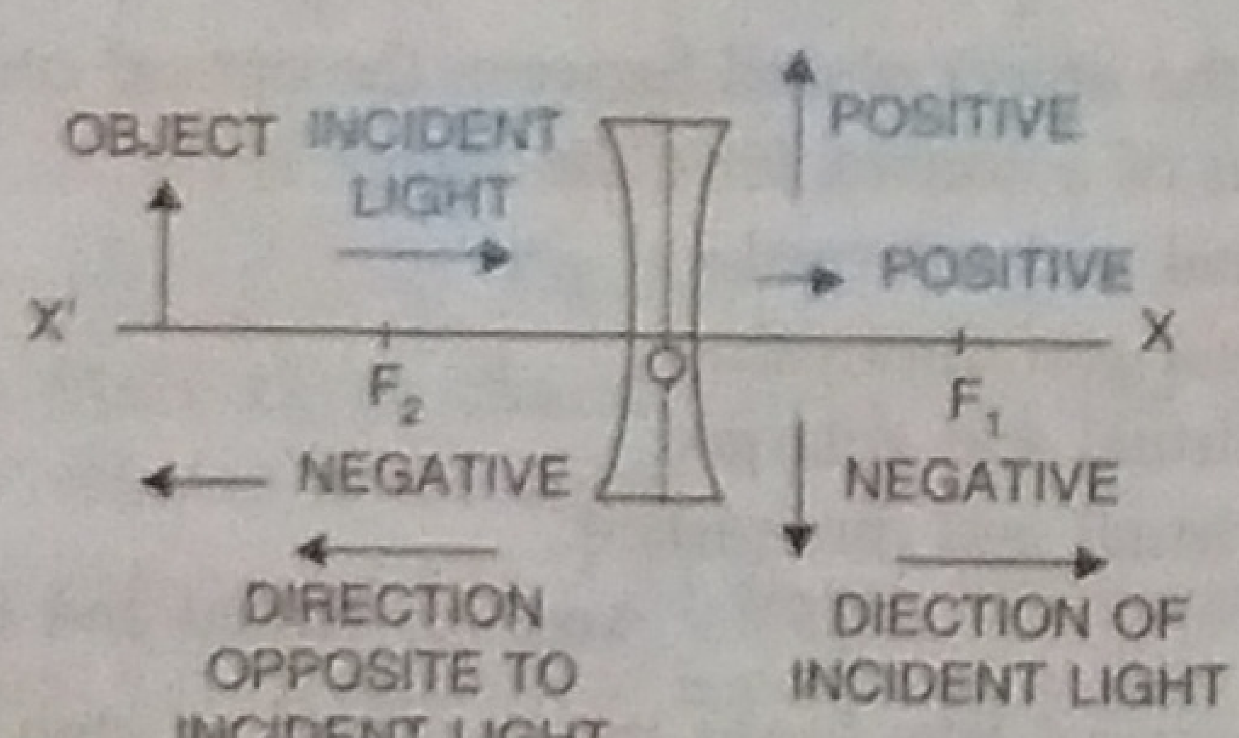
(2) The object is considered to be placed on the principal axis to the left of the lens.

(3) All the distances are measured along the principal axis from the optical centre of the lens. The distance of an object from the lens is denoted by u, the distance of image by v and the distance of second focus (*i.e.*, focal length of lens) by f.

(4) The distances measured in the direction of incident ray are taken positive, while the distances opposite to the direction of incident ray are taken negative.



(a) Convex lens



(b) Concave lens

Fig. 5.52 Sign convention

(5) The length above the principal axis is taken positive, while the length below the principal axis is taken negative.

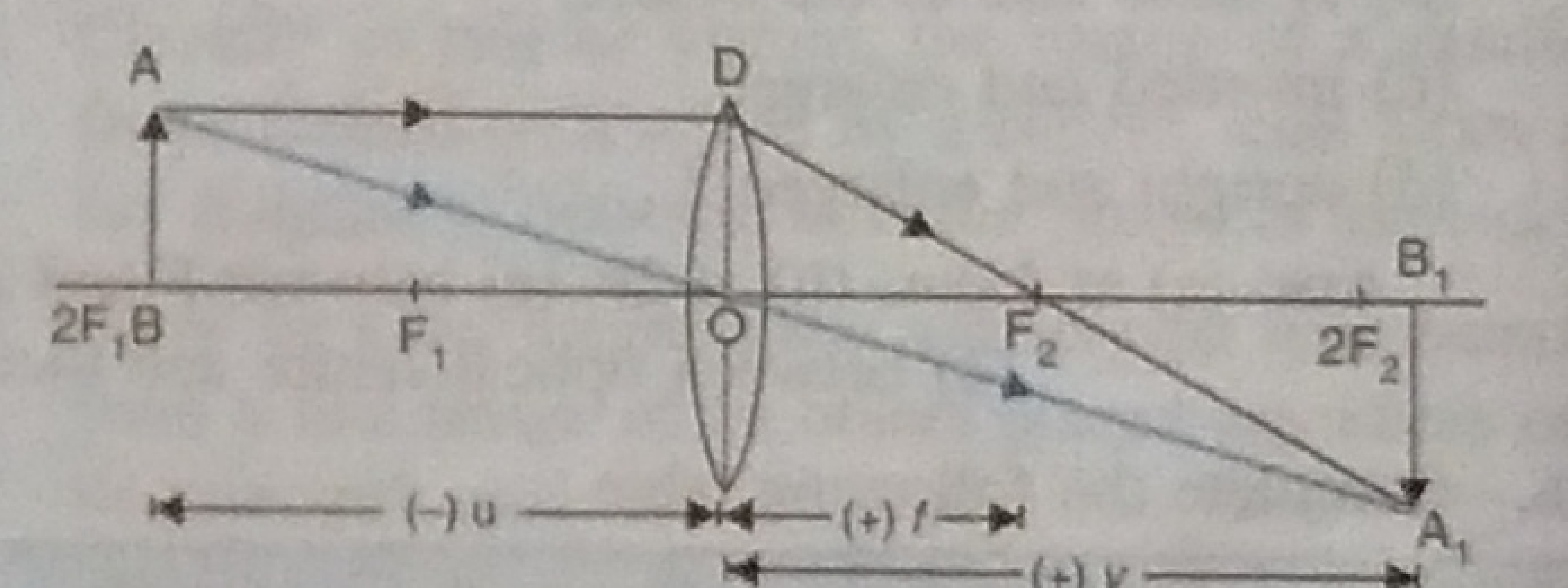
(6) Fig. 5.52 shows the distances for the convex and concave lens according to the sign convention. By sign convention, the *focal length* (= OF_2) *of the convex lens is positive and that of the concave lens is negative.* The distance of object (u) in front of lens is always negative. The distance of image (v) is positive if it is *real* and formed behind the lens, while it is negative if the image is *virtual* and formed in front of the lens.

5.11 LENS FORMULA $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

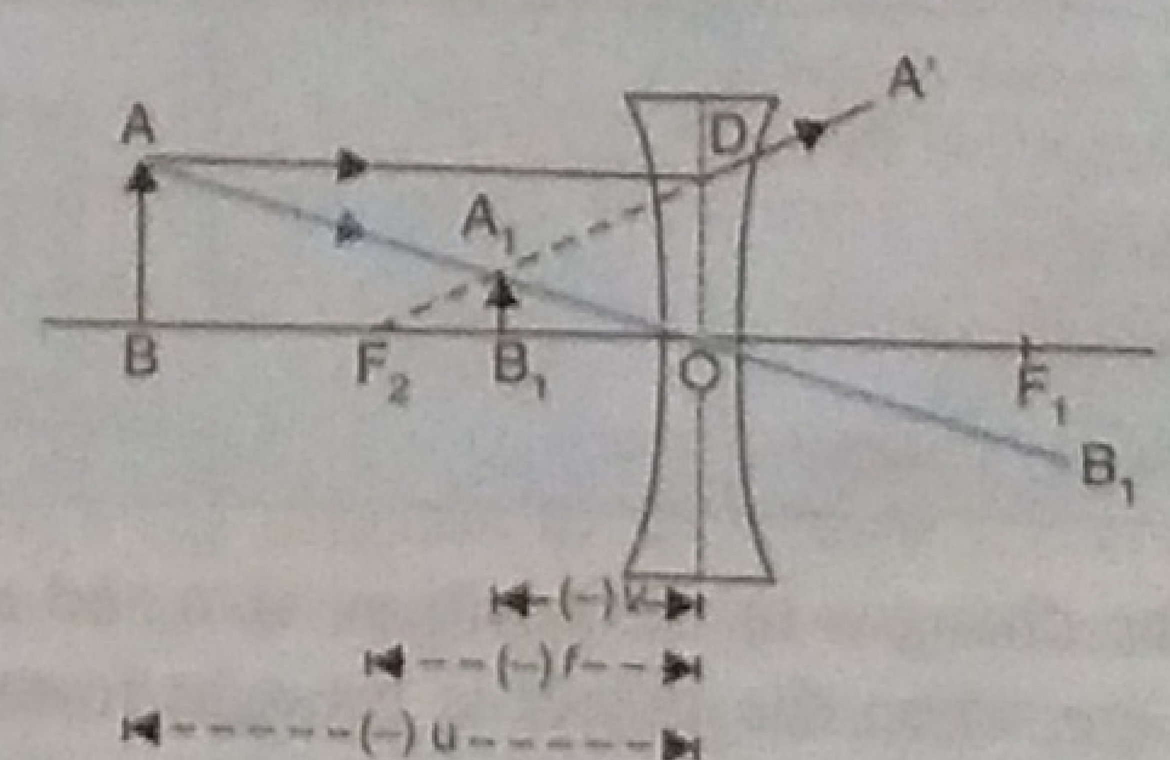
The equation relating the distance of object (u), distance of image (v) and focal length (f) of a lens is called the lens formula. It is same for both the convex and concave lens and is given as

$$\boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}} \quad \text{....(5.1)}$$

Derivation* : Fig 5.53 (a) and (b) show the formation of image A_1B_1 of a linear object AB by a convex and concave lens respectively.



(a) Convex lens



(b) Concave lens

Fig. 5.53 Lens formula

* Derivation is not included in the syllabus.

In Fig. 5.53, the triangles AOB and A_1OB_1 are identical

$$\therefore \quad \frac{BA}{B_1A_1} = \frac{OB}{OB_1} \quad ....(i)$$

Similarly $\Delta\ DOF_2$ and $\Delta\ A_1B_1F_2$ are identical

$$\therefore \quad \frac{OD}{B_1A_1} = \frac{OF_2}{F_2B_1}$$

But BA = OD

$$\therefore \quad \frac{BA}{B_1A_1} = \frac{OF_2}{F_2B_1} \quad ....(ii)$$

From eqns. (i) and (ii),

$$\frac{OB}{OB_1} = \frac{OF_2}{F_2B_1} \quad ....(iii)$$

(a) For convex lens : In Fig. 5.53(a), by sign convention, distance of object from the optical centre OB = u (negative), focal length of lens $OF_2 = f$ (positive), distance of image from the optical centre $OB_1 = v$ (positive), and

$F_2B_1 = OB_1 - OF_2 = v - f$ (positive)

$\therefore$ From eqn. (iii),

$$\frac{-u}{v} = \frac{f}{v-f}$$

or $\quad -uv + uf = vf$

or $\quad uf - vf = uv$

Dividing both sides by uvf,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad ....(iv)$$

(b) For concave lens : In Fig. 5.53(b), by sign convention, OB = u (negative), $OF_2 = f$ (negative), $OB_1 = v$ (negative),

and $\quad F_2B_1 = OF_2 - OB_1$

$\quad = f - v$ (negative)

Substituting these values in eqn. (iii),

$$\frac{-u}{-v} = \frac{-f}{-(f-v)} \quad \text{or} \quad \frac{u}{v} = \frac{f}{f-v}$$

or $\quad uf - uv = vf$

or $\quad uf - vf = uv$

Dividing both sides by uvf,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad ....(v)$$

From eqns. (iv) and (v), it is clear that the expressions are same for both the convex and concave lens.

> *Note :* In numericals, the known values are substituted with their proper sign and then the unknown quantity is obtained with its proper sign. According to sign convention for a convex lens u is always negative, f is always positive, v is positive for the real image and v is negative for the virtual image. But for a concave lens u, v and f all are negative and the numerical value of u is always greater than v.

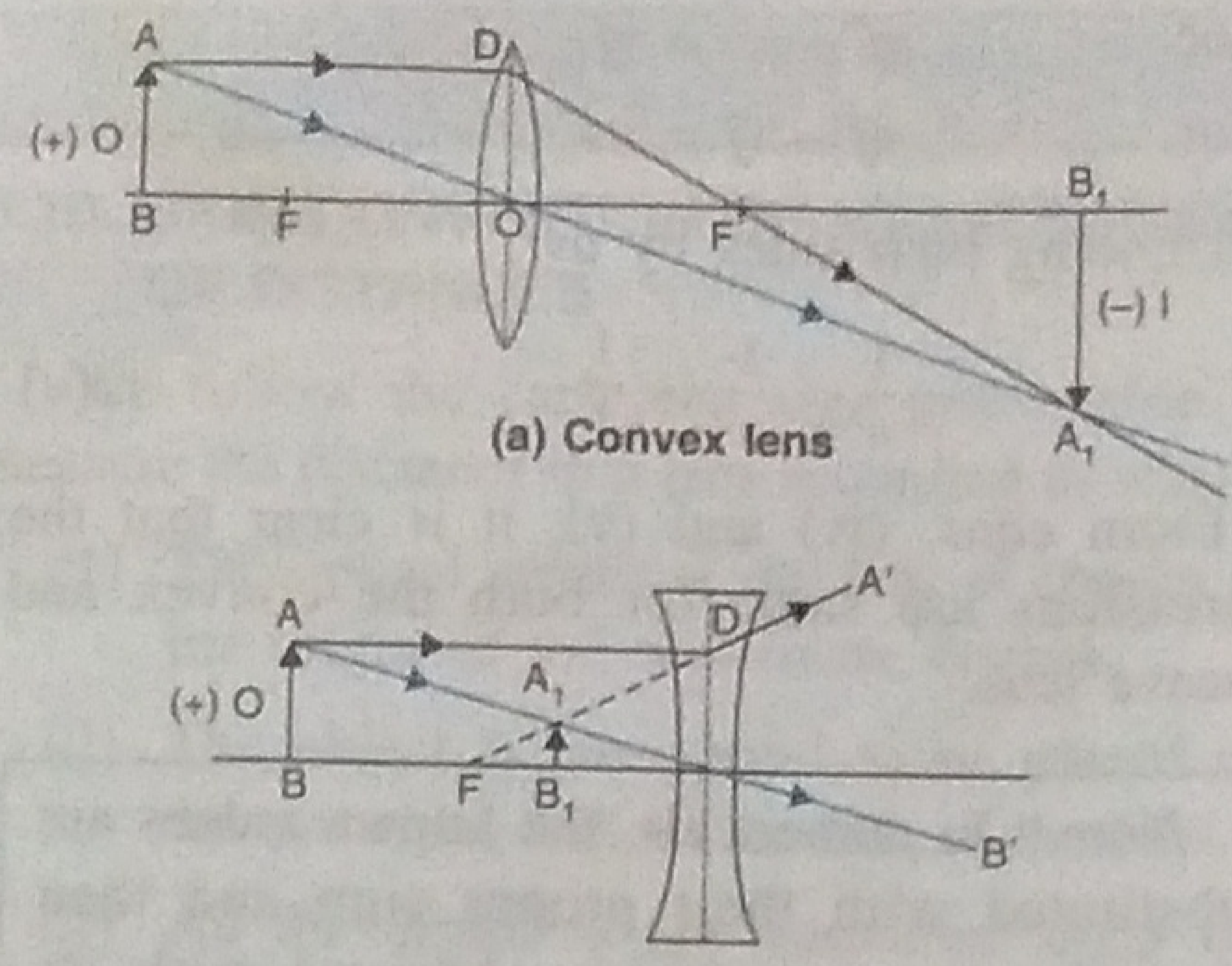
5.12 LINEAR MAGNIFICATION

It is not sufficient to know the postion of image of an object by a lens, but it is also required to know its size. When the position of object changes, there is a change in the position as well as the size of the image. *The ratio of length of image I perpendicular to the principal axis, to the length of object O, is called the linear magnification.* It is generally denoted by the letter m and is related to the distance of image (v) and distance of object (u) for both the convex and concave lens as :

> Linear magnification
>
> $$m = \frac{\text{length of image (I)}}{\text{length of object (O)}} = \frac{v}{u} \quad ...(5.2)$$

***Derivation :** Fig. 5.54 (a) and (b) show the formation of image A_1B_1 of an object AB by the convex and concave lens respectively.

* Drivation is not included in the syllabus



(a) Convex lens

(b) Concave lens

Fig. 5.54 Magnification

In Fig. 5.54, ΔABO and ΔA_1B_1O are similar triangles

$$\therefore \quad \frac{B_1A_1}{BA} = \frac{OB_1}{OB} \quad \text{....(i)}$$

(a) **For convex lens** : In Fig. 5.54(a), by sign convention, $B_1A_1 = I$ (negative), $BA = O$ (positive), $OB_1 = v$ (positive) and $OB = u$ (negative).

∴ From eqn. (i),

$$\frac{-I}{O} = \frac{v}{-u} \quad \text{or} \quad \frac{I}{O} = \frac{v}{u} \quad \text{....(ii)}$$

(b) **For concave lens** : In Fig. 5.54 (b), by sign convention, $B_1A_1 = I$ (positive), $BA = O$ (positive), $OB_1 = v$ (negative) and $OB = u$ (negative),

∴ From eqn. (i),

$$\frac{I}{O} = \frac{-v}{-u} \quad \text{or} \quad \frac{I}{O} = \frac{v}{u} \quad \text{....(iii)}$$

From eqns. (ii) and (iii), it is clear that the expressions for magnification are same for both the convex and concave lens.

Note : (1) For the real image (which is inverted), the magnification m is negative, while for the virtual image (which is erect), the magnification m is positive. Thus a convex lens can have the value of m positive as well as negative, but a concave lens always has the value of m positive.

(2) The numerical value of m is greater than 1 if image is magnified, is 1 for image of size same as of the object and is less than 1 for the diminished image. Thus the numerical value of m is always less than 1 for a concave lens, while it can be greater than, equal to or less than 1 for a convex lens depending upon the position of the object.

5.13 POWER OF A LENS

When a beam of light passes through a lens, it gets deviated from its path. The deviation produced by the lens is expressed in terms of its power. A lens which produces more deviation has more power. Thus,

The deviation of the incident light rays produced by a lens on refraction through it, is a measure of its power.

A thick lens *i.e.*, a lens (having surfaces of more curvature) is of short focal length and it deviates the rays more, while a thin lens (*i.e.*, a lens having surfaces of less curvature) is of large focal length and it deviates the rays less. Hence power of a lens is expressed (or measured) in terms of the reciprocal of its focal length. Its unit is **dioptre** (symbol D). Thus

$$\text{Power of lens (in D)} = \frac{1}{\text{focal length (in metre)}} \quad \text{..(5.3)}$$

While giving prescription to a patient, an optician does not quote the focal length of lens, but he quotes the power of lens. A lens is of power 1 dioptre (or 1 D), if its focal length is 1 m (or 100 cm).

Depending on the direction in which a lens deviates the light ray, its power is either positive or negative. If a lens deviates a ray towards its centre, the power is positive and if it deviates the ray away from its centre, the power is negative. Therefore the power of a convex lens is positive and of a concave lens is negative. Thus power of a convex lens of focal length 20 cm is + 5·0 D while that of a concave lens of same focal length is – 5·0 D.

If two thin lenses are placed in contact, the combination has a power equal to the algebraic sum of the powers of the individual lens. If a convex lens of power + 2·0 D is kept in contact with a concave lens of power – 2·0 D, the combination will have zero power and it will behave like a glass plate.

EXAMPLES

1. **An object of height 4·0 cm is placed at a distance 24 cm in front of a convex lens of focal length 8 cm. (a) Find the position and size of the image. (b) State the characteristics of the image.**

Given : O = 4·0 cm (positive), u = 24 cm (negative), f = 8 cm (positive)

(a) From relation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$

or $\frac{1}{v} = \frac{1}{-24} + \frac{1}{8} = \frac{1}{12}$

or $v = 12$ cm

The image is at distance 12 cm behind the lens.

From relation $\frac{I}{O} = \frac{v}{u}$, $\frac{I}{4 \cdot 0} = \frac{12}{-24}$

or $I = -\frac{12}{24} \times 4 = -2$ cm

Thus the image is inverted of size 2 cm.

(b) **Characteristics of the image :** The image is real, inverted and diminsihed (size 2·0 cm).

2. **The focal length of a camera lens is 20 cm. Find how far away from the film must the lens be set in order to photograph an object located at a distance 100 cm from the lens.**

Given : f = 20 cm (positive), u = 100 cm (negative)

From relation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$

or $\frac{1}{v} = \frac{1}{-100} + \frac{1}{20} = \frac{1}{25}$

or $v = 25$ cm

Thus the lens must be set at a distance 25 cm from the film towards the object.

3. **A convex lens forms an image 16·0 cm long of an object 4·0 cm long kept at a distance 6 cm from the lens. The object and the image are on the same side of lens.**

   (a) **What is the nature of image ?**

   (b) **Find : (i) the position of image, and (ii) the focal length of lens.**

(a) Since the image is magnified and on same side of the lens as the object, so the image is virtual.

(b) Given : I = 16·0 cm (positive), O = 4·0 cm (positive), u = 6 cm (negative)

(i) From relation, $m = \frac{I}{O} = \frac{v}{u}$, $\frac{16 \cdot 0}{4 \cdot 0} = \frac{v}{-6}$

or $v = -24$ cm

Thus the image is at a distance 24 cm in front of lens.

(ii) From realtion $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{f} = \frac{1}{-24} - \frac{1}{(-6)} = \frac{-1}{24} + \frac{1}{6} = \frac{1}{8}$

or $f = 8$ cm

Thus the focal length of lens = 8 cm.

4. **An object is placed at a distance of 10 cm in front of a concave lens of focal length 10 cm. Find :**

   (a) the position of image, and

   (b) the size of image in relation to the object.

Given : u = 10 cm (negative), f = 10 cm (negative)

(a) From relation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$

or $\frac{1}{v} = \frac{1}{-10} + \frac{1}{-10}$ or $\frac{1}{v} = -\frac{1}{5}$ or $v = -5$ cm

Thus the image is formed at a distance 5 cm in front of the lens.

(b) From relation $\frac{I}{O} = \frac{v}{u}$, $\frac{I}{O} = \frac{-5}{-10} = \frac{1}{2}$

Thus the size of image is half the size of object.

5. **Where must an object be placed in front of a convex lens of focal length 20 cm to obtain a real**