# KRISHNAGAR ACADEMY <br> HALF-YEARLY (PHASE - II) EXAM' 2020-2021 <br> CLASS -XI SUB. - MATHS F.M. - 60 

## Attempt all the questions from the Group-A and either Group-B or Group-C <br> GROUP-A (50 Marks)

Q. $1 a) R$ is a relation on $Z$ defined by $R=\{(a, b): a, b \in Z, a-b$ is an integer $\}$.

Find the domain and range of R .
b) Prove that $\tan \left(\frac{\pi}{3}+x\right) \tan \left(\frac{\pi}{3}-x\right)=\frac{2 \cos 2 x+1}{2 \cos 2 x-1}$
c) If $\cos (\theta+\varphi)=m \cos (\theta-\varphi)$, then prove that $\tan \theta=\frac{1-\mathrm{m}}{1+\mathrm{m}} \cot \varphi$.
d) If $x+i y=\left(\frac{1+\mathrm{i}}{1-\mathrm{i}}\right)^{3}-\left(\frac{1-\mathrm{i}}{1+\mathrm{i}}\right)^{3}$, then find $(\mathrm{x}, \mathrm{y})$.
e) If the sum of the certain number of terms of the A.P. $25,22,19, \ldots \ldots$ is 116 , find the last term.
f) The first term of a G.P. is 729 and $7^{\text {th }}$ term is 64 . Find the sum of first 7 terms of the G.P.
g) If the lines $2 x+y-3=0$ and $5 x+k y-3=0$ and $3 x-y-2=0$ are concurrent, find the value of k .
Q. 2 Express the function $f: A \rightarrow Q$ such that $f(x)=\frac{2 x-1}{3 x+2}$, where $A=\{x: x \in Z,|x| \leq 4\}$ as the set of ordered pairs and hence find its range.
Q. 3 If $\tan (\theta+\varphi)=m \tan (\theta-\varphi)$, then prove that $\sin 2 \theta=\frac{m+1}{m-1} \sin 2 \varphi$
Q. 4 If $x \cos \theta=y \cos \left(\theta+\frac{2 \pi}{3}\right)=z \cos \left(\theta+\frac{4 \pi}{3}\right)$,then find the value of $x y+y z+z x$.

## OR

If angle $\theta$ is divided into two parts such that the tangent of one part is $k$ times the tangent of the other, and $\varphi$ is their difference, then show that $\sin \theta=\frac{\mathrm{k}+1}{\mathrm{k}-1} \sin \varphi$
Q. 5 If $(x+i y)^{\frac{1}{3}}=a+i b$, where $x, y, a, b \in R$, show that $\frac{x}{a}-\frac{y}{b}=-2\left(a^{2}+b^{2}\right)$.
Q.6 In an A.P., if $\mathrm{p}^{\text {th }}$ term is $\frac{1}{\mathrm{q}}$ and $\mathrm{q}^{\text {th }}$ term is $\frac{1}{\mathrm{p}}$, prove that the sum of first pq terms

$$
\begin{equation*}
\text { is } \frac{1}{2}(p q+1) \tag{4}
\end{equation*}
$$

Q. 7 Find the value of $n$ so that $\frac{a^{n+1}+b^{n+1}}{a^{n}+b^{n}}$ be the geometric mean between $a$ and $b$. [4]

OR
How many words can be formed by using the letters of the word INDEPENDENCE in which
i) the words start with P
ii) the words begin with I and end wih P.
Q. 8 Using Principle Mathematical Induction, prove that
$1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots \cdots+\frac{1}{1+2+3+\cdots \cdot+n}=\frac{2 n}{n+1}$.
Q. 9 Prove that the coefficient of $x^{n}$ in the expansion of $(+x)^{2 n}$ is twice the coefficient of $x^{n}$ in the expansion of $(1+x)^{2 n-1}$.

OR
Show that the middle term of the expansion of $\left(x-\frac{1}{x}\right)^{2 n}$ is $\frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdots \cdots \cdots \cdot(2 n-1)}{n!}(-2)^{n}$.

## GROUP - B

## Attempt all questions

Q. 10 Find the coordinates of a point on the parabola $y^{2}=8 \mathrm{x}$ whose focal distance is 4 .

OR
Prove that the equation of the chord joining the points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ lying on the
Parabola $y^{2}=4 a x$ is $\left(y-y_{1}\right)\left(y-y_{2}\right)=y^{2}-4 a x$.
Q. 11 If l and $\mathrm{l}^{\prime}$ be the lengths of segments PS and $\mathrm{P}^{\prime} \mathrm{S}$ of a focal chord $\mathrm{PP}^{\prime}$ of a parabola

$$
\begin{equation*}
\mathrm{y}^{2}=4 \mathrm{ax}, \text { then show that } \frac{1}{1}+\frac{1}{1^{\prime}}=\frac{1}{\mathrm{a}} \text {. } \tag{6}
\end{equation*}
$$

## GROUP-C

## Attempt all questions

Q. 12 Calculate the median of the following frequency distribution:
$\begin{array}{llllllll}\text { Class limits: } & 130-134 & 135-139 & 140-144 & 145-149 & 150-154 & 155-159 & 160-164\end{array}$
$\begin{array}{cllllllll}\text { Frequency : } & 5 & 15 & 28 & 24 & 17 & 10 & 1\end{array}$

Find the $7^{\text {th }}$ decile $59^{\text {th }}$ percentile from the following frequency distribution:

| Marks | $:$ | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ |
| :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| No. of students: | 20 | 32 | 40 | 46 | 38 | 30 | 28 | 16 |  |

Q. 13 The following is a frequency distribution:
[6]

| Marks | $: 30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ | $80-90$ | $90-100$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

$\begin{array}{llllllll}\text { No. of students: } & 3 & 4 & 13 & 24 & 40 & 30 & 6\end{array}$

## Find Mean, Median and Standard Deviation.

