KRISHNAGAR ACADEMY

HALF-YEARLY (PHASE – II) EXAM' 2020-2021

<u>CLASS – XI SUB. – MATHS F.M. – 60</u>

Attempt all the questions from the Group-A and either Group-B or Group-C

GROUP-A (50 Marks)

<u>Q.1</u> a) R is a relation on Z defined by $R = \{(a, b): a, b \in Z, a - b \text{ is an integer}\}.$ [7×2]

Find the domain and range of R.

b) Prove that $\tan\left(\frac{\pi}{3} + x\right) \tan\left(\frac{\pi}{3} - x\right) = \frac{2\cos 2x + 1}{2\cos 2x - 1}$

c) If
$$\cos(\theta + \phi) = m\cos(\theta - \phi)$$
, then prove that $\tan \theta = \frac{1-m}{1+m} \cot \phi$.

- d) If $x + iy = \left(\frac{1+i}{1-i}\right)^3 \left(\frac{1-i}{1+i}\right)^3$, then find (x, y).
- e) If the sum of the certain number of terms of the A.P. 25, 22,19, is 116, find the last term.
- f) The first term of a G.P. is 729 and 7th term is 64. Find the sum of first 7 terms of the G.P.
- g) If the lines 2x + y 3 = 0 and 5x + ky 3 = 0 and 3x y 2 = 0 are concurrent, find the value of k.

[4]

<u>**Q.2**</u> Express the function $f: A \to Q$ such that $f(x) = \frac{2x-1}{3x+2}$, where $A = \{x: x \in Z, |x| \le 4\}$ as the set of ordered pairs and hence find its range.

Q.3 If
$$\tan(\theta + \phi) = m \tan(\theta - \phi)$$
, then prove that $\sin 2\theta = \frac{m+1}{m-1} \sin 2\phi$ [4]

Q.4 If
$$x \cos\theta = y \cos\left(\theta + \frac{2\pi}{3}\right) = z \cos\left(\theta + \frac{4\pi}{3}\right)$$
, then find the value of $xy + yz + zx$. [4]
OR

If angle θ is divided into two parts such that the tangent of one part is k times the tangent of the other, and ϕ is their difference, then show that $\sin\theta = \frac{k+1}{k-1}\sin\phi$

0.5 If
$$(x + iy)^{\frac{1}{3}} = a + ib$$
, where x, y, a, $b \in \mathbb{R}$, show that $\frac{x}{a} - \frac{y}{b} = -2(a^2 + b^2)$. [4]

<u>Q.6</u> In an A.P., if pthterm is $\frac{1}{q}$ and qth term is $\frac{1}{p}$, prove that the sum of first pq terms is $\frac{1}{2}(pq + 1)$. [4]

<u>Q.7</u> Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^{n}+b^{n}}$ be the geometric mean between a and b. [4]

OR

How many words can be formed by using the letters of the word INDEPENDENCE in which i) the words start with P ii) the words begin with I and end wih P.

Q.8 Using Principle Mathematical Induction, prove that

$$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots + \frac{1}{1+2+3+\dots+n} = \frac{2n}{n+1}.$$

Q.9 Prove that the coefficient of x^n in the expansion of $(+x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1 + x)^{2n-1}$. [6]

Show that the middle term of the expansion of $\left(x - \frac{1}{x}\right)^{2n}$ is $\frac{1.3.5.\dots(2n-1)}{n!}$ $(-2)^n$.

GROUP - B

Attempt all questions

Q. 10 Find the coordinates of a point on the parabola $y^2 = 8x$ whose focal distance is 4.

OR

Prove that the equation of the chord joining the points (x_1, y_1) and (x_2, y_2) lying on the

Parabola
$$y^2 = 4ax$$
 is $(y - y_1)(y - y_2) = y^2 - 4ax.$ [4]

Q. 11 If I and I' be the lengths of segments PS and P'S of a focal chord PP' of a parabola

$$y^2 = 4ax$$
, then show that $\frac{1}{l} + \frac{1}{l'} = \frac{1}{a}$. [6]

GROUP-C

Attempt all questions

Q.12 Calculate	[4]						
Class limits:	130-134	135-139	140-144	145-149	150-154	155-159	160-164
Frequency :	5	15	28	24	17	10	1

[6]

Find the 7 th decile 59 th percentile from the following frequency distribution:											
Marks	: 0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80			
No. of students:	20	32	40	46	38	30	28	16			
Q.13 The following is a frequency distribution:											
Marks :	30-40	40-50	50-60	60-70	70-8	0 8	80-90	90-100			
No. of students:	3	4	13	24	40)	30	6			
	1.0	1 1 5									

Find Mean, Median and Standard Deviation.